Modular localization and the holistic structure of causal quantum theory, a historical perspective

Dedicated to the memory of Jürgen Ehlers (1929-2008)

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Abstract

Recent insights into the conceptual structure of localization in QFT ("modular localization") led to clarifications of old unsolved problems. The oldest one is the Einstein-Jordan conundrum which led Jordan in

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1925 to the discovery of quantum field theory. This comparison of fluctuations in subsystems of heat bath systems (Einstein) with those resulting from the restriction of the QFT vacuum state to an open subvolume (Jordan) leads to a perfect analogy; the globally pure vacuum state becomes upon local restriction a strongly impure KMS state. This phenomenon of localization-caused thermal behavior as well as the vacuum-polarization clouds at the causal boundary of the localization region places localization in QFT into a sharp contrast with quantum mechanics and justifies the attribute "holstic". In fact it positions the E-J Gedankenexperiment into the same conceptual category as the cosmological constant problem and the Unruh Gedankenexperiment. The holistic structure of QFT resulting from "modular localization" also leads to a revision of the conceptual origin of the crucial crossing property which entered particle theory at the time of the bootstrap S-matrix approach but suffered from incorrect use in the S-matrix settings of the dual model and string theory.

The new holistic point of view, which strengthens the autonomous aspect of QFT, also comes with new messages for gauge theory by exposing the clash between Hilbert space structure and localization and presenting alternative solutions including a new look at old problems of actual interest as the "Schwinger-Higgs screening".

1 Preface

The subject of this paper grew out of many discussions with the late Jürgen Ehlers about Jordan's discovery of quantum field theory (QFT) and in particular the events between the publication of his thesis on statistical quantum mechanics in 1924 [1] and his discovery of QFT which was published in one section of the famous 1926 "Dreimännerarbeit" [2] together with Born and Heisenberg.

I met Jürgen Ehlers the first time around 1957 at the university of Hamburg when he was Jordan's assistant and played the leading role in Jordan's general relativity seminar. Our paths split, after I wrote my diploma thesis in particle theory and particle physics moved to the newly constructed high energy laboratory at DESY away from the old institute. Contacts with Ehlers and the relativity group were less frequent and ended when both of us took up research associate positions at different universities in the US. Only 40 years later, when he moved to Potsdam/Golm in the 90s as the founding director of the AEI and I was already close to retirement from the FU Berlin, we met a second time. At that time he was interested to understand some of Jordan's early work on quantum field theory for which he became famous[1]. In particular he was interested to understand some subtle points in the dispute between Jordan and Einstein's concerning Einstein's use of statistical mechanics fluctuation arguments in favor of the existence of photons which culminated in what is nowadays referred to as the *Einstein-Jordan conundrum.*

As the terminology reveals, the E-J conundrum was a poorly understood

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[1] After ww II Jordan interest was mainly focussed on general relativity and philosophical implications. As a result Jürgen and I were quite ignorant about his important work on QFT.

relation between fluctuations caused by restricting the vacuum state to the observables in a subvolume in Jordan's newly discovered field quantization and Einstein's use of statistical mechanics arguments which led him to identify a particle-like component in the fluctuation spectrum of a black body radiation gas with a corpuscular nature of light. In Ehlers's opinion [3], the foundational aspects of the behavior of the global vacuum upon restriction to observables localized in a subvolume were similar to problems about the origin of the cosmological constant. He hoped that with my experience of almost 40 years of work on problems of QFT I could be of some help. I learned recently through John Stachel that conjectures about possible connections between thermal aspects of the subvolume fluctuations in QFT with the Hawking-Unruh type problems already existed in the 80s [5].

For a some time this problem remained out of my range of interest, I did not want to loose time on something which would draw me into unclear historical problems away from my research concerning a nonperturbative access to QFT via "modular localization"[4]. But around 2007 I suddenly realized that the complete understanding of the E-J conundrum can be obtained with the help of precisely those new insights. One just had to apply the *principle of modular localization* which assigns a certain number of properties to localized subalgebras; the relevant property in this case being the thermal KMS[2] property of the impure state which results from the restriction of the global vacuum to the observables which were localized in a finite region. Those statistical mechanics fluctuation properties (which led Einstein to postulate the existence of photons) are indeed also present in the vacuum state if one only tests the state with the ensemble of observables which are localized in a (without loss of generality) causally complete region. This curious theoretical observation results from the causal localization principle and for this reason this effect is not realized in quantum mechancs (QM); in fact it is this counter-intuitive aspect with respect to the intuition of physicists (which in most cases came from QM) which later led to the term "conundrum". Nowadays we know that it is a predecessor of the Unruh effect. It is an irony of history that Jordan discovered QFT without ever becoming aware of a property which places QFT into its strongest conceptual contrast with QM. This makes the E-J conundrum and its solution, as well its relation to other counter-intuitive aspects of QFT and, last not least, the role of the underlying causal localization principle in the actual research the most fascinating topic of the history of QFT.

When I wanted to explain my findings [7][1][19] in 2008 to Ehlers, I learned that he passed away shortly before my return from Brazil to Berlin.

The main aim of this paper, which I dedicate to the memory of Jürgen Ehlers, is to explain my findings and their relation to other open problems in QFT in more detail as in [7]. One of these problems, which Ehlers in his capacity as the founding director of the AEI took an interest in, was string theory (ST). He was annoyed by the fact that he was unable to bridge the gaps between his

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[2] The analytic characterization of thermal equilibrium states which survives in the thermodynamic limit when the Gibbs trace formula is lost [6]. KMS states cannot be described in terms of density matrix and the limiting von Neumann algebra changes its type.

understanding of spacetime properties and gravity and the (sometimes bizarre) claims of members of the ST group at the AEI. What kept his interest alive was his lifelong curiosity and the obvious considerable mathematical effort as well as the substantial reputation of the protagonists of these new ideas.

The work on modular localization also led me to string-localized fields and their important improved short distance property which promised a radical extension of renormalization theory to interaction between fields with higher spins. The reason why I mention this here is that this new concept of string-localization also revealed that string theory (ST) and its derivatives (embedding, dimensional reduction, properties of "branes") had nothing to do with string localization in spacetime but was rather the result of a fundamental misunderstanding of causal localization. Hence in a curious way Ehlers problems with the ancient Einstein-Jordan conundrum and his problems with ST were interconnected. His death in 2008 prevented me from revealing this new message.

It is the purpose of these notes to explain its constructive [7] as well as critical [9] power in a historical context. Usually a historical paper revisits the past about closed subjects; a typical example are research papers on the discovery and the conceptual struggle of QM. In contrast to such subjects which are closed from a foundational point of view, the situation of the problems addressed in this paper is very different in that they only were solved recently and that the theory in which they appeared is still far from its closure. Some of the new concepts which not older than one decade.

My posthumous thanks for introducing me to a fascinating topic from the genesis of QFT which, far from being a closed part of history, exerts its conceptual spell over actual particle theory, go to Jürgen Ehlers. The present exploration of the foundational principle of modular localization did not only change the view about hitherto incompletely understood problems at the dawn of QFT [7], but also promises to have an important say about its future [9].

2   Introduction

A dispute between Einstein and Jordan (referred to as the E-J conundrum [10]) led Jordan to propose the first quantum field theoretical model in order to show that there exists a quantum analog of Einstein's thermal fluctuations in open subvolumes in form of two-dimensional quantized Maxwell waves in a global ground state. A brief sketch of the pre-history which led Jordan is essential:

- Einstein (1909, more details 1917 in [11]): calculation of mean square fluctuations in an open subvolume in statistical mechanics of black body radiation shows two components: wave- and particle-like ("Nadelstrahlung") which Einstein interpreted as intrinsic pure theoretical evidence for photons (in addition to the observational support coming from the photoelectric effect).

- Jordan in his PhD thesis (1924, [12]) argued that the particle-like component $\sim \bar{E}_\nu h\nu$ is not needed for attaining equilibrium.

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- Einstein's reaction [13] consisted in the statement that Jordan's argument seems to be mathematically correct but physically flawed (the absorption is incorrectly described). He praised Jordan's statistical innovations ("Stosszahlansatz").

- Einstein's paper caused Jordan's radical change of mind; he fully accepted Einstein's view by demonstrating that he can obtain the same wave- and particle-like fluctuation components by restricting a "two-dimensional quantized Maxwell field" (modern terminology: d=1+1 chiral current model) to a subinterval. In this way he discovered field quantization probably without understanding *why* a vacuum in QFT behaves radically different from a quantum mechanical vacuum.

Shortly after this episode Jordan published his first field quantization in a separate section in the famous 1926 "Dreimännerarbeit" [2]. Gaps in Jordan's computation and his somewhat artistic treatments of infinities caused some ruffling of feathers with his coauthors Born and Heisenberg [10]. From a modern point of view the picture painted in some historical reviews, namely that this was a typical case of a young brainstorming innovator set against a scientific establishment (represented by Born), is not quite correct. Born and Heisenberg had valid reasons to consider Jordan's fluctuation calculations as incomplete, to put it mildly; conceding this does however not lessen Jordan's merits as the discoverer of QFT .

One reason why this discovery of QFT was not fully embraced at the time was that, although a free field on its own (staying with its linear properties) is a quite simple object, the problem of energy fluctuations in open subvolumes is anything but simple; to understand why subvolume fluctuations in ground state problems of QFT are similar to Einstein's statistical mechanics thermal fluctuations is a deep conceptual problem which cannot be solved solely by calculations, especially because such calculations can only be done in terms of conceptually uncontrolled approximations. but it can be satisfactory answered with the help of advanced ideas which relate the restriction of the vacuum to the observables of a spacetime subvolume with thermal properties and vacuum polarization ("split inclusions" of modular localized algebras [6]). One may safely assume that Born and Heisenberg perceived that this new field model of Jordan with infinitely many oscillators did not quite fit into their continuation of the quantum mechanical project which Heisenberg started a short time before; in particular Jordan's nonchalant way of handling infinities led to critical comments [10].

Nevertheless Heisenberg, who in comparison to Jordan understood very little about statistical at the time of the E-J conundrum, probably discovered vacuum polarization (which is absent in QM) under the influence Jordan's fluctuation problem. A letter he wrote to Jordan before he published his famous vacuum polarization paper [10], he challenges Jordan to account for a logarithmic divergence $lim_{\varepsilon \to \infty} ln\varepsilon$, $\varepsilon =$ fuzziness at the interval ends (next section). Indeed vacuum polarization and thermal manifestations of localization are opposite sides of the same coin.

One note of caution. Since the terminology particles and waves played an important role in the Einstein-Jordan dispute, the reader may think that the E-J conundeum is related to the quantum mechanical particle-wave duality (which was solved at the time of Bohr in terms of different but equivalent descriptions of QM). This is not the case, rather the conundrum referers to the counterintuitive fact that the vacuum in QM is inert, whereas in QT with causal localizability as QFT (but not relativistic QM) it turns into a statistical mechanics-like impure KMS state on spatially localized subalgebras.

The important distinction between the global quantum mechanical nature of infinitely many oscillators and their holistic use in the implementation of causal localization in a quantum theory of local fields had to wait almost 5 decades before being understood on a foundational level. For some time QFT was suspected to be afflicted by internal inconsistencies which lead to ultraviolet divergencies; even after discovering the covariant renormalized perturbation theory for quantum electrodynamics and finding an impressively successful agreement of low order perturbation with experimental observations, some of these doubts lingered on. Renormalized perturbation theory remained for a long time a collection of recipes about how to extract time-ordered correlation functions from the quantization rules starting with classical Lagrangians.

This quantization parallelism to the classical field theory of Faraday and Maxwell as embodied in the Lagrangian or functional integral quantization prevented for a long time an awareness about some radical differences of the quantum counterpart of Einstein's formulation of relativistic causality in Minkowski space. One such difference was that quantum fields, in contrast to smooth causally propagating classical functions, were rather singular operator-valued Schwartz distributions which required testfunction smearing in order to attain the status of (generally) unbounded operators with which one then can form operator algebras which are causally localized in spacetime regions. The other surprise was that these operator algebras have properties which were somewhat unexpected from the conceptual viewpoint of QM in that causal localization causes the global vacuum state to become impure upon restriction to a local operator subalgebra $\mathcal{A}(\mathcal{O})$ generated by covariant fields $A(x)$ smeared with $\mathcal{O}$-supported test functions. These impure "partial" states fulfill the so-called KMS property [6] with respect to a *modular Hamiltonian* which is intrinsically determined by the pair $(\mathcal{A}(\mathcal{O}), \Omega_{vac})$ of local algebra and vacuum state vector. The mathematical theory of operator algebras which highlights such properties is the *Tomita-Takesaki modular operator theory* which is omnipresent in QFT thanks to its causal localization structure. The presentation of QFT in terms of a net of operator algebras and their properties was proposed by Rudolf Haag [14] shortly after Arthur Wightman published his characterization of covariant fields in terms of properties of their correlation functions [15] which afterwards entered the first modern textbook in which a new axiomatic view about QFT was formulated. Haag's textbook [6] on "local quantum physics" (LQP) based on an operator-algebraic approach to QFT appeared only many decades after he gave a first account of this new formulation [14]. The terminology LQP in the present article is used whenever it is important to remind the reader that the

arguments go beyond the view about QFT he meets in most textbooks which is mostly restricted to a formulation of perturbation theory within the setting of Lagrangian quantization and its functional integral formulation.

The mathematical property which guaranties the applicability of this theory is the *standardness* of the pair $(\mathcal{A}(\mathcal{O}), \Omega_{vac})$ i.e. the property that the operator algebra acts on Ω_{vac} (more generally on all finite-energy state vectors) in a cyclic $(\overline{\mathcal{A}(\mathcal{O})\Omega_{vac}} = H)$ and separating ($\mathcal{A}(\mathcal{O})$ contains no annihilators of Ω_{vac}) manner. The first property is a result of the presence of a positive energy representation of the Poincare group (the Reeh-Schlieder property [6]), whereas the second property results from spacelike commutativity of observables and is equivalent to the fact that also the commutant which contains the algebra of the causal complement $\mathcal{A}(\mathcal{O})' \supsetneqq \mathcal{A}(\mathcal{O}')$ acts cyclic on Ω_{vac}, as long as the spacelike complement $\mathcal{O}'$ is non-void. It is these properties which are the cause why, despite the shared attribute of being quantum theories, QFT is radically different from QM [16].

For a structural comparison of the two quantum theories it is convenient to rewrite (the Schrödinger form of) QM into the Fock space setting of "second quantization". In this formulation the newly introduced vacuum remains, different from its active role in QFT, completely inert with respect to the action of the Schrödinger "quantum field" (no vacuum polarization); instead of the cyclic action the local algebra at a fixed time[3] corresponding to a spatial region $\mathcal{R} \subset \mathbb{R}^3$, one obtains a subspace and a tensor factorization of H

$$H(\mathcal{R}) = \overline{\mathcal{A}(\mathcal{R})\Omega_{inert}} \subset H = H(\mathcal{R}) \otimes H(\mathcal{R}^\perp) \tag{1}$$
$$\mathcal{A}(\mathcal{R}) = B(H(\mathcal{R})),\ \mathcal{A} \equiv \mathcal{B}(\mathcal{H}) = \mathcal{A}(\mathcal{R}) \otimes \mathcal{A}(\mathcal{R}^\perp)$$

of This inertness of the quantum mechanical vacuum is very different from the "vacuum polarizability" of Ω_{vac} in QFT which in turn is connected to the lack of tensor factorization (despite the commutation between $\mathcal{A}(\mathcal{O})$ and $\mathcal{A}(\mathcal{O}')$). In terms of structural properties of operator algebras these remarkable differences in the mathematical structure amount to the existence of two non-isomorphic factor algebras in QFT: the global $\mathcal{B}(H)$ algebra of all bounded operators on a Hilbert space and the local *monad* algebras $\mathcal{A}(\mathcal{O})$ which are all isomorphic to the unique hyperfinite type III$_1$ factor algebra in the Murray-von Neumann-Connes classification of factor algebras [6]. The present choice of terminology reveals the intention to see the new local quantum physical view of QFT in analogy to the way Leibnitz understood reality in terms of relations between monads. In this extreme relational view a monad by itself is structureless similar to a point in geometry. Indeed in the local quantum physical description of QFT all properties of quantum matter, including the Poincare covariance of its localization in spacetime as well as the localization-preserving inner symmetries in which it is localized, can be shown to arise from modular positioning of copies of the abstract monade within a shared Hilbert space (section 3).

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[3] In LQP such an algebra at a fixed time $\mathcal{A}(\mathcal{R})$ is defined as the intersection of all spacetime algebras $\mathcal{A}(\mathcal{O})$ with $\mathcal{R} \subset \mathcal{O}$.

Together with the thermal KMS property of the locally restricted vacuum there is the formation of a vacuum polarization cloud at the causal boundary of localization which accounts for a *localization entropy*. By replacing the boundary by a thin shell of size ε the localization entropy can be described in terms of a (dimensionless) function of the dimensionless area $\alpha = area/\varepsilon^2$ which diverges in the limit $\varepsilon \to 0$. This relation between the increasing sharpness of localization and the increasing localization entropy is the *substitute of the lost Heisenberg uncertainty relation in QM*. The position operator $\mathbf{x}_{op}$ is a global quantum mechanical observable which does not belong to the observables obeying the causal localization principle of LQP. The divergence in the sharp localization limit $\varepsilon \to 0$ also shows another aspect in which QFT differs from QM. The entanglement between the wedge-localized algebra and its opposite (that of the spacelike separated wedge) is always infinite in the sense that it is not possible to describe the associated state as density matrix; indeed there are no pure states nor density matrix states on monad algebras; all states are impure in a very radical way. In quantum statistical mechanics such states appear as KMS states in the thermodynamic limit of Gibbs states (density matrices).

Reduced vacuum states assign a probability to the ensemble of local observables contained in $\mathcal{A}(\mathcal{O})$; with other words the notion of probability is intrinsic to quantum causal localization; unlike the probability interpretation which Born added to QM and which led to heated epistemological philosophical discussions about how the quantization of a theory of an individual mechanical observable requires a probabilistic interpretation and led Einstein to reject quantum mechanics as a final description of reality (the Dear Lord does not throw dice). But the intrinsic probability coming from quantum causal localization is not different from that of statistical mechanics since it refers to the ensemble of all observables which share the property of being localized in the same region. The same KMS identity is shared by all observables which are localized in the same spacetime region[4]. In fact this statistical mechanics-like probability is precisely the kind of probability in Einstein's used use of the fluctuation properties of black-body gas. The understanding *why* this is also present in Jordan's model of QFT in the vacuum state was the aspect which was missing for a solution of the conundrum. In retrospection QFT could have appeared together with the probability of localized ensembles right from the beginning and not only Einstein's livelong resistance against assigning probabilities to individual quantum events[5] but also the whole course of particle theory could have taken a different direction right from the beginning!

But in the real world such big conceptual jumps are virtually impossible; even for getting from inertial systems in Minkowski spacetime to General Relativity it took Einstein many years and the same can be said about the development

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[4] The *causal completion property* of LQP permits to assume without loss of generality that the region is causally closed..

[5] In QFT each individual local observable inherits the KMS property of the local ensemble to which it belongs. Perhaps with a future more foundational understanding about the relation of QM with QFT it will be possible to understand that the imagined ensembles in global QM are relics of real localized KMS ensembles of QFT.

of QM from the old semiclassical Bohr-Sommerfeld ideas. The problem for the case at hand is aggravated by the fact that up to the middle of the 60s there did not even exist a mathematical framework of operator algebras in which ideas about "modular localization" could have been formulated. It is interesting to note that modular operator theory (on the physical side often referred to a modular localization) is the only theory to whose discovery and development mathematicians (Tomita, Takesaki) and physicists (Haag, Hugenholz and Winnink) contributed on par. They first realized this at a 1965 conference in Baton Rouge[6] with statistical mechanics of open systems and the role of the KMS property representing the physical side [6]. The study of the relation between modular operator theory and causal localization in LQP started a decade later [17], and its first application consisted in a more profound understanding [18] of the Unruh Gedankenexperiment [20]. The terminology "modular localization" is more recent and marks the beginning of a new constructive strategy in QFT based on the modular aspects of localization of states and algebras [43][4].

The E-J conundrum represents in fact a precursor of the Unruh Gedankenexperiment and, as the latter, can be fully resolved in terms of the principle of modular localization. In fact in the special case of Jordan's chiral current model on the side of QFT the E-J relation it represents a perfect unitary *isomorphism* between a system defined by the vacuum state restricted to the algebra $\mathcal{A}(I)$ localized in an interval I and an associated global statistical mechanics system at finite temperature. Such isomorphic relations are referred to as describing an "inverse Unruh effect", [23] and the Jordan model is the only known illustration. However in both cases the KMS temperature is not something which one can measure with a thermometer or use for "egg-boiling" and neither is the localization region filled with radiation (section 5).

On several occasions the attribute "holistic" will be used in connection with modular localization. This terminology has been previously applied by Hollands and Wald [24] in connection with their critique of calculations of the cosmological constant in terms of simply occupying global energy levels (with a cutoff at the Planck mass). In previous papers [25] as in the present work, it refers to the intrinsicness of localization which is connected with the cardinality of phase space degrees of freedom and their subtle local interplay which distinguishes physical localization of quantum matter from mathematical/geometrical concepts. In fact it presents a strong resistance against attempts of geometrization of QFT.

The simplest illustration of the meaning of holistic consists in the refutation of the vernacular: "(free) quantum fields are nothing more than a collection of oscillators" which often students are told after having taken a basic course of QM. Knowing continuous families of oscillators in the form of creation and annihilation operators $a^{\#}(\mathbf{p})$ does not reveal anything about free quantum fields

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[6]The mathematicians worked on the generalization of the modularity of Haar measures ("unimodular") in group representation theory whereas the physicists tried to understand quantum statistical mechanics directly in the thermodynamic infinite volume limit (open system statistical mechanics) by using the KMS identity instead of approaching this limit by tracial Gibbs states.

and their associated local operator algebras. The free Schrödinger field and a free scalar covariant field share the same global creation/annihilation operators

$$a_{QM}(\mathbf{x},t) = \frac{1}{(2\pi)^{\frac{3}{2}}} \int e^{i\mathbf{p}\mathbf{x}-\frac{\mathbf{p}^2}{2m}} a(\mathbf{p})d^3p, \ \ [a(\mathbf{p}),a^*(\mathbf{p}')] = \delta^3(\mathbf{p}-\mathbf{p}') \tag{2}$$

$$A_{QFT}(x) = \frac{1}{(2\pi)^{\frac{3}{2}}} \int \left(e^{-ipx}a(\mathbf{p}) + e^{ipx}a^*(\mathbf{p})\right) \frac{d^3p}{2\sqrt{\mathbf{p}^2+m^2}}, \ p = (\mathbf{p}, \sqrt{\mathbf{p}^2+m^2})$$

In both cases the global algebra is the irreducible algebra of all operators $B(H)$, generated by the shared creation/annihilation operators, but the local algebras[7] generated by test function smearing of the fields with finitely supported Schwartz functions $supp f(\mathbf{x}) \subset \mathcal{R}$ of the fields and its canonical conjugate at a fixed time in a spatial region $\mathcal{R}$ are very different. In the relativistic case they are identical to the algebras $\mathcal{A}(\mathcal{O}_{\mathcal{R}})$, $\mathcal{O}_{\mathcal{R}} = \mathcal{R}''$ i.e. the causal spacetime completion of $\mathcal{R}$ which is also generated by smearing with $\mathcal{O}_{\mathcal{R}}$-supported spacetime smearing functions. According to what was stated before these algebras are of monad type and the $\mathcal{A}(\mathcal{O}_{\mathcal{R}})$-restricted vacuum state is a KMS state; in the case of the Schrödinger field the associated subalgebra $B(H(\mathcal{R}))$ maintains the character of the global algebra; the vacuum continues to be an inertial state in the "smaller" factor Hilbert space $H(\mathcal{R})$.

Whereas the global QM algebra is simply the tensor product of its factor algebras, the relation of the net of local algebras to its $\mathcal{A}(\mathcal{O})$ "pieces" is a more holistic relation; although together with its complement it generates the global algebra $\mathcal{A}(\mathcal{O}) \vee \mathcal{A}(\mathcal{O})' = B(H)$, the global algebra $B(H)$ is not a tensor product of the two. The most surprising property which underlines the terminology "holistic" is the fact that the full net of local operator algebras which contains all physical informations can be obtained by "modular tuning" *of a finite number of copies of a monad* in a shared Hilbert space[8]; the reader who is interested in the precise formulation and its proof is referred to [26] also [16]. The fact that the global oscillator variables are the same in both cases (2) does not reveal these fundamental holistic differences which have very different physical consequences. The present quantization formalism (Lagrangian, functional integral) does not shed light on properties of QFT which solve the Einstein-Jordan conundrum in a clear-cut way. If it comes to ensemble properties of localized observables, the global aspects of covariant fields on which covariant perturbation theory is founded are of lesser importance than the local operator algebras $\mathcal{A}(\mathcal{O})$ which are generated by all smeared fields $A(f)$ with $\sup p f \subset \mathcal{O}$. The emphasis changes from covariance properties of fields to properties of relative localization of operattor algebras.

It is precisely this holistic aspect which renders any calculation of the subvolume fluctuation difficult, the simplicity of global oscillators is of no help here.

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[7] Technical points as the connection between fields and the algebras they generate are not important in the present context and therefore will be omitted.

[8] This number n is two for rhe simplest case of a chiral algebra, whereas for a net in four spacetime dimension the correct modular positioning can be achieved in terms of n=7 copies. The emergence of the spacetime symmetries in Minkowski spacetime as well as possible inner symmetries of quantum matter is a consequence of this holistic tuning.

A calculation in closed form is (even in the absence of interactions) not possible, and the imposition of covariance, which was the important step for obtaining the modern form of perturbation theory, does not provide any guidance. For renormalized perturbation theory one has clear recipes which were extracted from the imposition of covariance, but this is of not much help when one wants to find "good" variables for the description of localized fluctuation. Saying that the global aspects can be described in terms of oscillators is almost as useless for understanding QFT as trying to understand a living body in terms of its chemical composition. Although modular localization theory asserts the existence of modular Hamiltonians, in its present state it does not provide ways to construction them. Jordan's chiral model is an exceptional case for which, similar to the Unruh Gedankenexperiment, an explicit identification of the modular Hamiltonian in terms of the spacetime symmetries of the model is possible. Actually one may view Jordan's fluctuation problem as a predecessor of the Unruh effect: QFT was born with the "thermal"[9] localization aspects of the E-J conundrum which includes a completely intrinsic pre-Born notion of probability"; however the proximity of its date of birth to that of QT prevented an in-depth inestigation of differences and possibly shared properties beyond $\hbar$ and the Hilbert space.

This begs the question how, with the understanding of foundational properties of QFT still being incomplete, it was possible to achieve the remarkable progress in renormalized perturbation; or to phrase it a more historical context: how could one arrive at the standard model without having first solved the 1925 Einstein-Jordan conundrum? The answer is surprisingly simple: to get from the old Wenzel-Heitler formulation of perturbation theory, in which the vacuum polarization contribution were still missing, to the Tomonage-Feynman-Schwinger- Dyson perturbation theory for quantum electrodynamics (QED), one only needed to impose covariance and "exorcise" some ultraviolet divergences by finding plausible recipes and finally check the consistency of the so obtained prescription. Many years later there were also derivation of these renormalization rules by starting from invariant free field polynomials (without using Lagangian quantization[10]) and invoking spacelike commutativity in an inductive way (the causal perturbation setting of Epstein and Glaser [27]). But such conceptual refinements of reducing prescriptions to to an underlying principle had little impact and in any case would not have helped to obtain the foundational insight into modular localization which is required in order to solve the E-J conundrum.

This lucky situation of making progress by playfully pushing ahead and working once way through a yet incomplete formalism with the help of consistency checks did not extend beyond Lagrangian quantization and renormalized perturbation theory. As will be shown in section 6, it is precisely this setting which determined the fate of QFT for more than half a century which is now being replaced by a more general setting based on modular localization. The latter has

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[9] The reason for the quotation marks will be explained in section 6.
[10] The free fields do not have to fulfill Euler-Lagrange equations.

not only removed unnecessary restrictions from renormalization theory, but also led to a different view about on-shell constructions (section 5). When, in the aftermath of the Lehmann-Symanzik-Zimmermann (LSZ) scattering theory and the successfull adaptation of the Kramers-Kronig dispersion relations the first attempts of S-matrix based on-shell construction were formulated, the conceptual difficulties of analytic aspects of on-shell properties were underestimated. As one knows through more recent progress about modular localization, an important aspect of the S-matrix, namely its role as a relative modular invariant of wedge-localization was missing and as a result, the true nature of the particle crossing property was misunderstood by identifying it with Veneziano's dual model crossing which than led to string theory.

The correct formulation of the on-shell crossing property within a new S-matrix based construction project and the solution of the E-J conundrum are interconnected via the principle of modular localization. It is the aim of this paper to show the power of the latter by presenting the solution to these two problems. The first attempts to formulate particle physics and obtain an constructive access outside of quantization and perturbation theory was the S-matrix in Mandelstam's project [28]. As we know nowadays, and as it will be explained in detail in the present work, this failed as a result of the insufficient understood on-shell analytic properties, whose connection to the causality principles is more subtle than those to the off-shell correlation functions. In retrospect it is clear that with the scant understanding of the central crossing property and more generally the conceptual origin of on-shell analyticity properties, there was no chance in 70s for Mandelstam's S-matrix based particle theory project to succeed. In retrospect it is also clear why this happened precisely when Veneziano's mathematical construction of a crossing symmetric meromorphic function in two variables was accepted as a model realization of particle crossing for elastic scattering amplitudes. It is appropriate in an article whose intention is to shed historical and philosophical light to try to explain this situation in its historical context.

The importance of the E-J conundrum in the development of QFT can be best highlighted by following Galileio's example and invent a dialog between Einstein and Jordan which is based on the same facts about the subvolume fluctuation problem but takes place at the beginning of 1927 after Max Born added his probability interpretation to Heisenberg's and Schrödinger's quantum mechanics. Its invention is to argue that by a slight change of attention the development of QFT could have taken a very different course. Einstein who had just countered the claim in Jordan's thesis that his "Nadelstrahlung" (the particle-like component in Einstein's statistical mechanics subvolume fluctuation spectrum on which his ideas of photons was based) is unnecessary, whereupon Jordan not only withdrew his mathematical objections, but even tried to convince Einstein that the same two components appear in his first 2-dimensional model of a quantized field.

Einstein: Dr. Jordan, I appreciate that you could finally accepted my invitation to come to Berlin and I am extremely interested to understand why, after first criticizing my fluctuation calculations in my statistical mechanics

thermal blackbody radiation model in your Goettingen thesis, you now claim you can obtain not only the wave-like component but also my "Nadelstrahlung-component in your wave quantization even at zero temperature.

Jordan: Thank you Professor Einstein for taking so much interest in my work. The appearance of such a fluctuation spectrum in my new setting of quantized waves in a vacuum state is indeed surprising because my model uses quantization rules which are generalizations of those in Heisenberg's quantum mechanics. But even if one would formulate QM (as you did in your pre-Heisenberg quasiclassical statistical mechanics calculations on black body radiation) as a thermal many body system, this would not change the fact that the ground state remains inert. A quantum mechanical system in the ground state cannot produce thermal-like fluctuations by restricting it to a subvolume. Despite similarities in the quantization procedure, the fluctuations in a localized subsystem cannot be thermal-like. My quantized Maxwell waves cannot be subsumed into a quantum mechanics of systems with a large particle number.

Einstein: As you remember, I have some grave reservation against associating a probability to an individual measurement on a quantized mechanical system which I expressed in the formulation "the Dear Lord does not throw dice", which meanwhile became a vernacular. But I never had any problem with probability in statistical mechanics, in fact my calculation of the Nadelstrahlung-component in the black body fluctuation spectrum, which led me to the particle nature of light, is based on the probability of statistical mechanics. Does the result of your subvolume fluctuation calculation in the ground state of your field quantization mean that this state does not remain inert if looked upon from a local point of view ?

Jordan: Professor Einstein, I am glad that you raised this question. I have been breaking my head over these unexpected consequences of my new quantized field theory and I would be dishonest with you, if I claim to understand these conceptual implications. But since the main difference to mechanics is the causal propagation, which was already implicit in the Nahewirkungsprinzip of Faraday and Maxwell and which you then succeeded to generalize into your new relativity principle in a Minkowski spacetime, I am inclined to suspect that the ensemble aspect which one needs in order to avoid the assignement of a probability to an individual mechanical system as proposed by my adviser Prof. Max Born has its origin in the quantum realization of causal localization. Somehow this principle creates a natural ensemble associated with its causal completion of a localization region, namely the ensemble of all local observables attached to that spacetime region. I tried to convince Prof. Born and my colleague Heisenberg with whom I am presently collaborating on a joint publication, but they conceded a separate chapter to me only with reservations. It would be very helpful for me to obtain some support from your side.

Einstein: I need some time to think about this new situation. Your conjecture means that the new theory of quantum fields, which is certainly much more fundamental than Heisenberg's and Schrödinger's quantized mechanics, comes with an intrinsic notion of localized ensembles of observables and the associated statistical mechanics type of probability. If one would better understand how

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the less fundamental global quantum mechanics can be related as a limiting case to your new fundamental quantum field theory in such a way that Born's added probability is a relict of the ensemble probability (which your analogy of subvolume fluctuations suggests to be an intrinsic property of the quantum counterpart of causal localization), I may have to modify my rejection of Born's probability and perhaps even my Weltanschauung. Let us remain in contact and please keep me informed about future clarifications about the points raised in out conversation.

In this imagined dialog which could have radically changed the history of QFT, I tried to not use the concept of modular localization (there was no mathematical support in the 20s) but limit the ideas to those which Unruh had at his disposal (free fields and analytic thermal aspects of a spacetime restricted vacuum expectation values).

The organization of this paper is as follows. In the next section the vacuum polarization on the boundary of causal localization is derived for the "partial charge", which is a modern formulation of Heisenberg's original observation. Section 3 sketches the issue of modular localization and its KMS property with special emphasis on the difference between a KMS temperature and that measured by a thermometer. In section 4 the KMS property is used for the explicit construction of an isomorphism between the thermal subvolume (interval in Jordan's chiral model) fluctuations in Jordan's model with a corresponding statistical mechanics model representing Einstein's side. Section 5 explains modular localization and its relation with the Tomita-Takesaki modular operator theory. The impact of modular localization on on-shell constructions of QFT with particular emphasis on the connection of particle crossing with the KMS identity is addressed in section 6. The perhaps most important consequence of modular localization for the ongoing research in particle theory is the generalization of renormalized perturbation to interactions involving arbitrarily high spin through the use of string-localized fields in section 6. But in the same section the origin of the misunderstandings which led to the dual model and ST are also explained. Section 8 addresses some little known relations between the cardinality of phase space degrees of freedom and localization which also includes a critique of the Maldacena conjecture concerning the nature of the AdS-CFT correspondence. Other arguments which expose localization as the Achilles heel of ST were already presented in section 7. It is the aim of the present work to address in particular those problems which cannot be solved without foundational knowledge about modular localization.

3 Vacuum polarization, area law

In 1934 Heisenberg [29] finally published his findings about v. p. in the context of conserved currents which are quadratic expressions in free fields. Whereas in

QM they lead to well-defined partial charges associated with a volume V,

$$\partial^\mu j_\mu = 0, \ Q_V^{clas}(t) = \int_V d^3x j_0^{clas}(t,\mathbf{x}) \tag{3}$$

$$Q_V^{QM}(t) = \int_V d^3x j_0^{QM}(t,\mathbf{x}), \ Q_V^{QM}(t)\Omega^{QM} = 0$$

there are no sharp defined "partial charges" Q_V in QFT, rather one finds (with g_T a finite support smooth interpolation of the delta function) [30]

$$Q(f_{R,\Delta R}, g_T) = \int j_0(\mathbf{x},t)f_{R,\Delta R}(\mathbf{x})g_T(t)d\mathbf{x}dt, \ f_{R,\Delta R} = \begin{pmatrix} 1, \ \|x\| \leq R \\ 0, \ \|x\| \geq R + \Delta R \end{pmatrix}$$

$$\tag{4}$$

$$lim_{R\to\infty}Q(f_{R,\Delta R}, g_T) = Q, \ \|Q(f_{R,\Delta R}, g_T)\Omega\| = \begin{cases} F_2(R,\Delta R) \overset{\Delta R\to 0}{\sim} C_2 ln(\frac{R}{\Delta R}), \ n = 2 \\ F_n(R,\Delta R) \overset{\Delta R\to 0}{\sim} C_n(\frac{R}{\Delta R})^{n-2}, \ n > 2 \end{cases}$$

The *dimensionless* partial charge $Q(f_{R,\Delta R}, g_T)$ depends on the "thickness" (fuzziness) $\Delta R = \varepsilon$ of the boundary and becomes the f and g-independent (and hence t-independent i.e. conserved) global charge operator in the large volume limit. The deviation from the case of QM are caused by vacuum polarizations (v. p.). Whereas the latter fade out in the $R \to \infty$ limit, they grow with the dimensionless area $\frac{R}{\Delta R}$ for $\Delta R \to 0$. The simplest calculation is in terms of the two-point function of conserved current of a zero mass complex scalar free field. In the massive case the leading term in the limit $\Delta R \to 0$ remains unchanged. We leave the elementary calculations (not elementary at the time of Heisenberg) to the reader.

The presence of vacuum polarization causes relativistic quantum fields to be more singular than Schrödinger fields and require the formulation in terms of Schwartz distribution theory for which the above smearing of the current with smooth finitely supported test function serves as an illustration. The LQP setting on the other hand avoids the direct use of such singular objects in favor of local operator algebras. In such a description the singular nature of vacuum polarization does not appear directly in the individual operators but rather shows up in ensemble properties of operator algebras. It turns out that under rather general conditions there exists between two monad algebras a distinguished (by modular theory) intermediate type I_∞ algebra $\mathcal{N}$ [6]

$$\mathcal{A}(\mathcal{O}_{\mathcal{R}+\Delta R}) \supset \mathcal{N} \supset \mathcal{A}(\mathcal{O}_{\mathcal{R}}), \ H \overset{V}{\to} H \otimes H, \ \eta \equiv V(\Omega \otimes \Omega) \tag{5}$$

$$VAB'\Omega = A\Omega \otimes B\Omega, \ A \in \mathcal{A}(\mathcal{O}_{\mathcal{R}}), B' \in \mathcal{A}(\mathcal{O}_{\mathcal{R}+\Delta R}), \ V\mathcal{N}V^* = B(H) \otimes \mathbf{1}$$

The algebra $\mathcal{N}$ is simply a $B(H)$ in the first factor of the factorization. In QM the V would be simply the identity operator consistent with the trivial tensor factorization of the vacuum.

However the QFT vacuum does not split on $\mathcal{N} \otimes \mathcal{N}'$ in this trivial manner, since the operator V generates an entanglement which is uniquely determined

in terms of the state vector η which in turn is uniquely related to the inclusion. Different from entanglements in quantum information theory, *one does not have to average* over the degrees of freedom in the second factor, rather the density matrix which corresponds to the global vacuum is simply obtained by restricting the vacuum state to $\mathcal{N}$. The split vacuum is a rather involved state which is canonically defined in terms of the inclusion and the global vacuum which has nontrivial components to all higher particle states which are allowed by superselection rules. In the limit $\Delta R \rightarrow 0$ the type one factor $\mathcal{N}$ converges to the monad $\mathcal{A}(\mathcal{O}_{\mathcal{R}})$,and the tensor factorization is lost. In this limit the entanglement becomes very singular which is reflected in the fact that the $\mathcal{N}$-restricted density matrix Gibbs state becomes a more singular KMS state.

The strength of the entanglement is measured in terms of the von Neumann entropy of the $\mathcal{N}$-associated density matrix obtained by restricting the global vacuum state to $\mathcal{N}$. From the above effect of the vacuum polarization cloud on the dimensionless partial charge with a fuzzy surface of size ΔR one would expect that the vacuum polarization cloud within the split distance behaves in the same way. This leads to an dimensionless area law for the localization entropy (LE)

$$LE(a) \sim a, \ a \equiv A/(\Delta R)^2, \ for \ \Delta R \rightarrow 0 \tag{6}$$

a result which is also supported by 't Hooft's brickwall picture [31]. For chiral theories which live on a lightray, the divergence is $LE(a) \sim lna, \ a = I/\varepsilon$ for $\varepsilon \rightarrow 0$, where I is the length of an interval on the lightray and ε is a small interval around the endpoints of I; this result follows from the fact that Jordan's model in the E-J conundrum is an isomorphism which relates the standard heat bath statistical mechanics behavior with the localization entropy (section 4). Although all objects, including the distinguished $\mathcal{N}$ algebra and the splitting operator V, are determined in terms of the *standard inclusion* $(\mathcal{A}(\mathcal{O}_{\mathcal{R}+\Delta R}) \supset \mathcal{A}(\mathcal{O}_{\mathcal{R}}), \Omega)$ [11], it is presently very difficult to do explicit computations in the as yet insufficiently developed modular operator setting.

There is another idea which leads to a logarithmically modified area law for entropy which is based on the analogy with the dimensionless volume proportionality of the heat bath entropy. This picture suggests to think in terms of "box" in which two sides are space-like and the third one is lightlike. The two-dimensional case has no transverse direction; the contribution from the lightlike direction is equal to the known logarithmic behavior. It suggests a logarithmically modified area law

$$LE(a) \sim alna, \ \Delta R \rightarrow 0 \tag{7}$$

This result would place the localization entropy in close relation with the heat bath entropy; the thermodynamic entropy would be related to the localization

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[11] A standardness for a pair $(\mathcal{A}(\mathcal{O}), \Omega)$ means that Ω is cyclic and separating for $\mathcal{A}(\mathcal{O})$ which follows from the Reeh-Schlieder theorem [6]. Standardness of an inclusion means that the two included algebras and the relative commutant of the smaller within the bigger are standard with respect to Ω.

entropy by a change of parametrization which account for the change of a space-like with a lightlike direction (with the dimensionless volume $\sim V(kT)^3$). The thermodynamic limit algebra is a monad as the sharply localized algebra, but the parametrizations in the approach of the two monads by sequences of $B(H)$ algebras is different. This behavior of the localization entropy would correspond to the existence of a correspondence between the localization entropy and the conventional heat bath entropy at a certain temperature (a weak kind of "inverse Unruh effect" [23]).

The final answer about the leading terms in the localization entropy has to await the improvements of computational techniques within modular operator theory. The ultraviolet divergences of localization entropy in the sharp localization limit is a consequence of the principle of causal localization and in no way indicates a threat against the existence of QFT; it already occurs in the absence of interactions. It does however show that the intuitive content of causal localization is lost even if one only wants to realize it in form of a thought-experiment with existing hardware; already the Unruh effect shows that the observer (thermometer, counter) has to be uniformly accelerated i.e. one has to go outside inertial systems and for double cones instead of noncompact wedges the required conformal counters to be placed onto conformal trajectories simply do not exist. In this connection it is important to note that there are two kinds of temperatures which coalesce in inertial systems but differ in Unruh situations and (as a result of the Einstein equivalence principle) in black hole situations.

Haag's formulation of QFT in terms of nets of local observable algebras is based on the quantum adaptation of the causal localization as formulated in Einstein's use of the Minkowski spacetime (with physical roots in the Nahewirkungsprinzip of Faraday and Maxwell [6]) is clear on a conceptual/intuitive level, but, as explained before leads to problems in terms of hardware. However a certain intuitive appeal coupled with a precise mathematical formulation is all one can ask for in the realm of principles of quantum physics; the latter should lead to observational consequences but need not be directly observable. In later sections we will meet experimentally accessible consequences of causal localization. For their derivation it is helpful to reformulate causal localization in a way which is independent of the use of particular "field coordinatizations". This is achieved in the algebraic setting of LQP. For reasons which become clearer in the next section, this intrinsic formulation is referred to as *modular localization*. The entropical aspect corresponds to the thermal KMS manifestation of modular localization. In particular in the case of black holes, when the localization horizon is not the "fleeting" kind of observer-dependent construct but an intrinsic property of the spacetime metric, the localization entropy corresponds precisely to the KMS thermal manifestation (the Hawking "temperature") of the part of the world which is localized outside the black hole horizon.

Note that the well known entropy conjecture by Bekenstein, based on equating a certain area behavior in classical General Relativity with entropy results formally from the above area law by equating ΔR with the Planck length. Quantum Gravity is often thought of that still elusive theory which explains why and how the quanta of gravity can escape the consequences of modular localization

for sharp localization. If Bekenstein's conjecture is valid and the possibility that gravity is (like van der Waals forces) an effective residue of fundamental quantum matter is excluded, the conclusion that modular localization has to be replaced by a more foundational principle is unavoidable.

The relation between ΔR and the entropy is reminiscent of Heisenberg's quantum mechanical uncertainty relation in which the uncertainty in the position is replaced by the split distance ΔR within which the vacuum polarizations can attenuate so that outside the vacuum returns to play its usual role if tested with local observables in the causal complement of $\mathcal{O}_{\mathcal{R}+\Delta R}$.

It was already mentioned in the introduction and will be stressed here again that the probability interpretation, which Born had to add to Heisenberg's and Schrödinger's formulation of QM, is completely intrinsic to LQP. It is a consequence of the "thermal" KMS property of ensembles of operators contained in a localized algebra $\mathcal{A}(\mathcal{O})$ in the $\mathcal{O}$-restricted vacuum state. It is in no way different from the statistical mechanic probability, which Einstein used in his fluctuation arguments to show the particle character of photons, and by which he challenged the physical content of Jordan's thesis. It is only with the modern concept of modular localization and the hindsight of more than 8 decades that one realizes how close the E-J conundrum scraped past the intrinsic probability coming from the quantum formulation of the Faraday-Maxwell-Einstein causal locality principle in Minkowski spacetime. QM is, as classical mechanics, a global theory of individual instantaneously interacting material bodies. It is conceivable that within a future more foundational understanding of a conceptual connection of local QFT with global QM, quantum probability survives even though modular localization is lost. Instead of a real ensembles the surviving probability refers to a fictitious ensemble which leaves its trace in the statistics of the outcome of repetitive experiments.

With Einstein in the possession of these implications of his dispute with Jordan, his philosophical stance with respect to quantum mechanical probabilities may have taken a different turn. and many philosophical discussions from the beginnings of QM up to the present time may have taken a different turn.

A particular radical illustration of this point is the reconstruction of a net of operator algebras from the relative modular position of a finite number of copies of the monad [16]. For chiral theories on the lightray one needs two monads realized in a shared Hilbert space in the position of a modular inclusion, for d=1+2 this "modular GPS" construction needs three and in d=1+3 six positioned monads [26] to create the full reality of a quantum matter world including its Poincaré symmetry (and hence Minkowski spacetime) from abstract modular groups as well as inner symmetries via the DHR superselection theory applied to the net of observable algebras (which determines the kind of quantum matter). This possibility of obtaining concrete models by modular positioning of a finite number of copies of an abstract monad is the strongest "holistic outing" of QFT. For d=1+1 chiral models the modular positioning leads to a partial classification of chiral theories as well as to their explicit construction (section 5).

Apart from d=1+1 factorizing (integrable) models, where such modular as-

pects were used for the existence proof, QFT has not yet reached the state of maturity where such holistic properties can be applied for classifications of families of models and their construction through controlled approximation. An extension to curved spacetime would be very interesting; the simplest question in this direction is the modular construction of the local diffeomorphism group on the circle in the setting of chiral theories.

4 Modular localization and its thermal manifestation

The aim of this section is to collect some properties which form the nucleus of LQP, among them "modular localization" as the intrinsic formulation of causal quantum localization. Since subalgebras in QFT $\mathcal{A}(\mathcal{O})$ localized in spacetime regions $\mathcal{O}$ are known to act cyclic and separating on the vacuum (the Reeh-Schlieder property [6]), the conditions for the validity of the Tomita-Takesaki modular theory are always fulfilled for regions whose causal completion is smaller than the Minkowski spacetime. For such regions QFT secures the existence of the uniquely defined Tomita operator $S_{\mathcal{O}}$ whose polar decomposition yields the building blocks of modular localization theory.

It has been known for a long time that the algebraic structure underlying free fields allows a functorial interpretation in which operator subalgebras of the global algebra $B(H)$ are the functorial images of *subspaces of the Wigner wave function spaces* ("second quantization"[12]).

LQP generalizes the quantization approach of QFT in the sense that it avoids the Lagrangian or functional integral parallelism to classical field theory; being more fundamental than classical field theory, the content of QFT should reveal itself without the classical detour. In contrast to QM, QFT in the LQP setting de-emphasizes individual operators in QFT in favour of *ensembles of operators* which share the same spacetime localization region. This intends to follow more closely the situation in the laboratory where the experimentalist measures coincidences between events in spacetime; all the rich particle properties, including the nature of spin and internal quantum numbers were obtained by repetitions and refinements of observations based on counters which are placed in compact spatial region and remain "switched on" for a limited time. Their detailed internal structure is generally not known, what matters is their localization in spacetime and the sensitivity of their response.

The role of covariant quantum fields in LQP is that of generators of a net of local operator algebras $\{\mathcal{A}(\mathcal{O})\}_{O \in R^4}$ which act in a fixed Hilbert space. In the Wightman setting a field is a covariant operator-valued distribution $A(x)$ which is globally defined for all $x \in R^4$. From its global definition one passes to (unbounded) $\mathcal{O}$-localized operators, formally written as $A(f) = \int A(x)f(x)d^4x$, $suppf \subset \mathcal{O}$, which according to Wightman's axioms, define a system of polyno-

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[12] Not to be confused with quantization; to quote a famous saying by Ed Nelson: "quantization is an art, but second quantization is a functor".

mial (unbounded) operator *-algebras $\mathcal{P}(\mathcal{O})$. Formally these unbounded operators can be associated with an aforementioned net of easier manageable bounded operators (von Neumann algebras) which define Haag's LQP setting. The advantage is that one obtains access to the well-developed mathematical theory of operator algebras (from now on omitting "bounded"). Certain causality aspects allow a more natural definition and more profound understanding in the LQP setting. The mathematical details which allow to pass from Wightman's description to LQP and vice versa are tedious and incomplete [6], but the lack of precise understanding about the mathematical connection had little effect on progress and plays certainly no role in a description of the historical and actual impact of concepts as modular localization and its holistic manifestations.

Whereas both settings are different formulations of closely related physical concepts, there is a significant distinction between these settings and constructions based on (Lagrangian/functional integral) quantization methods, where quantization has the well-known meaning of a canonical formalism which is build on a kind of parallelism to classical fields. Quantization is not a physical principle; whereas it is conceivable that certain successful classical descriptions of nature can be pictured as limiting cases of certain quantum theories, there is no general correspondence. The fact that the less fundamental QM (it lacks causal localization and its holistic consequences) is capable to maintain an almost (up to ordering prescriptions of operators) unique relation to classical mechanics does not imply that such a close relation must continue to hold in QFT. The strong link between classical mechanics and its quantum counterpart finds its best known expression in the fact that Lagrangian quantization (canonical quantization) and functional quantization (path integrals) enjoy solid mathematical support frrom measure theory.

All this breaks down in interacting QFT. Although one can formally start from such representation of quantization, there is no basis for a mathematically controlled procedure; the only thing which one can do is to impose the successful rules of renormalized perturbation theory; the functional representation, although without mathematical support, takes care of the correct perturbative combinatorics and other formal aspects. With the hindsight of covariance and plausible rules to remove infinities for renormalizable couplings, one finally arrives at a (divergent) perturbative series which, although in many cases observational successful, reveals nothing about the mathematical *existence* of a solution; hence the question whether the observational successful lowest order terms can be considered as an asymptotic approximation for vanishing interaction strength remains unanswered. It should be mentioned that in d=1+1 there exists a superrenormalizable class of interaction (free field like short distance behavior, no uncontrolled infinities) for which an extension of the measure theoretical path integral approach of QM permits a mathematical construction [32], but these models shed no light on realistic strictly renormalizable interactions.

The so-called *causal perturbation* setting of Epstein and Glaser [27] avoids running into the infinities which one encounters in the Lagrangian or functional setting; it has the additional advantage that it does not require Euler-Lagrange fields but rather works for all spinorial/tensorial fields which result from the co-

variantization of Wigner's positive energy representations (i.e. it is not a quantization setting [13]); this is of particular importance in case one uses stringlike covariant fields (section 6). Causal perturbation theory starts with a pointlike Wick-odered coupling of free fields to a composite scalar and implements the principle of causal commutativity in an inductive way by viewing the iterative step as a distributional extension problem, subject to a "minimal scaling" requirement which can only be fulfilled for (by definition) renormalizable couplings. For interaction densities whose short distance scaling properties remain within the so-called power counting limit (4 in d=1+3), one finally arrives at the renormalized perturbation series for operator-valued distributions (or their correlation functions) which depend on a finite number of coupling parameters.

There is a finite number of renormalizable interactions between massive scalar and s=1/2 fields; non of these couplings develops infrared divergencies in the massless limit. Interactions with s=1 vectormesons are on the borderline: the charge-neutral (gauge-invariant) quantities define a renormalizable subset, but the (physical) matter fields coupled to the vectormesons remain nonrenormalizable; only by passing from a Hilbert space setting to an indefinite metric Krein space, one can recover renormalizability in terms of unphysical but renormalizable fields; however the Krein formalism has not led to a prescription for extracting physical pointlike *matter fields*. In contradistinction to the $s < 1$ couplings, all vectormeson couplings have infrared divergencies for vanishing vectormeson masses. This behavior, as well as similar problems for higher spins, will be explained in the section 6 where it is also shown how to avoid it by the appropriate use of string-localization. The divergence of the perturbative series even after having renormalized each term, may be a result of the fact that, in contradistinction to QM, one is not perturbing operators but rather singular operator-valued distributions. LQP avoids the use of these singular objects in its operator-algebraic formulation; but apart from integrable systems and some not yet tested proposals for general systems, an approximation scheme in terms of operators instead of operator-valued distributions is still not available (section 7).

Thanks to the better short distance property of stringlike free fields, renormalized perturbation theory leads to an extension of the Hilbert space setting of renormalized perturbation theory to fields for any spin (section 6). In this way the number of couplings below the power-counting limit (the prerequisite of renormalizability) is enlarged from the finite number of pointlike couplings to an infinite number of couplings involving stringlike fields; this is a vast area for future investigations. A helpful analogy for the passing of pointlike generators to nets of algebras is that of changing from coordinate description of geometry to its intrinsic presentation. The net of operator algebras does not depend on which of the infinitely many possible field coordinatizations was used, only conserved currents associated to spacetime- and inner- symmetries maintain a preferential status. In fact the local net point of view de-emphasizes the

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[13]Free fields are obtained from the unique (m,s) positive energy Wigner representation of the Poincare group [44] either by (highly non-unique) covariantization or (for string-localized fields) by implementing modular localization [42].

role of individual operators by deriving the local net and all physical properties from the relative modular positioning of a *finite number of algebras* in a shared Hilbert space (a "GPS" formulation).

Even the spacetime symmetries and the nature of the spacetime on which they act, as well as internal symmetries are encoded in the *modular positioning*. As already mentioned in the introduction, the ensemble aspect of operators sharing the same spacetime localization region is not something added, but rather an intrinsic consequence of the LQP setting of QFT. In a way this setting is converse to the Atiyah-Witten project of the 70s in which operator properties were considered consequences of geometric properties.

The most conspicous physical manifestation of the LQP setting is the fact that the restriction of any finite energy pure state to a local algebra is *impure* on the ensemble of operators contained in that algebra; in fact the restriction is equal to a **K**ubo-**M**artin-**S**chwinger thermal state, whose main difference from a heat-bath state of statistical mechanics is that the Hamiltonian is not that of the global time translation but rather the *modular Hamiltonian* which is uniquely determined in terms of the local algebra $\mathcal{A}(\mathcal{O})$ and the global finite energy state. In case of the vacuum state and certain LQP models one can even identify this Hamiltonian with generators of covariances which leave $\mathcal{O}$ invariant; the Jordan model in the next section is one of such models. As mentioned on previous occasions, the statistical mechanics character of localized states in LQP comes with an ensemble probability, whereas in QM the probability interpretation has to be added.

The central issue in LQP refers to two physically motivated requirements on the local net

$$[\mathcal{A}(\mathcal{O}_1), \mathcal{A}(\mathcal{O}_2)] = 0, \ \mathcal{O}_1 >< \mathcal{O}_2, \ Einstein \ causality \qquad (8)$$
$$\mathcal{A}(\mathcal{O}) = \mathcal{A}(\mathcal{O}''), \ causal \ completeness$$
$$\mathcal{A}(\mathcal{O}') = \mathcal{A}(\mathcal{O})', \ Haag \ duality$$

The first line is a condensed notation for the commutativity of operators from spacelike separated regions; it is only required for observable fields. The commutation property for *nonabsorbable* operators, as those coming from spinor fields or fields carrying superselected charges, have commutation relations which are determined by the application of superselection theory to their associated observable subalgebras[14] [6]. The causal completeness (8) is a local adaptation of the old time-slice property [35]. Whereas causal commutativity refers to spacelike separations, causal completeness stands for the quantum counterpart of hyperbolic propagation of classical relativistic fields for which initial values at a fixed time determine the wave functions in the *causal shadow* of the initial values. The singular nature of quantum fields prevents a simple formulation in

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[14]Locality and the ensuing covariance in low spacetime dimensions allow more general commutation relations than Fermi/Bose relations. In d=1+2 particles and fields maybe plektons [33] (braid group statistics) and in d=1+1 the commutation relations can be changed within the same relative local field class (soliton class) without having any influence on particles ("schizons" [34]).

terms of fields and prefers the above clearer algebraic formulation. Interesting situations arise when Haag duality is violated; the simplest illustration is the Aharonov-Bohm effect for the net which is generated by the free electromagnetic field strength [36].

Mathematically it is very easy to construct Einstein-causal theories which violate causal completeness; as a consequence they have pathological physical properties[15]. Well-known cases result from generalized free fields with certain continuous mass distributions [35]. The $AdS - CFT$ correspondence and the construction of "branes" within a higher dimensional QFTs are also illustrations (section 7) for violations of causal completeness. On the other hand the holographic projection onto a null-hypersurface leads to the correct reduction of degrees of freedom so that its cardinality adjusts itself to the lower dimensional situation. As a consequence one cannot return to the original theory without some additional information.

As a result of a subtle relation between the cardinality of phase-space degrees of freedom with localization (split property, causal completeness,..), the nuclearity property, introduced decades ago by Buchholz and Wichmann [6], became in conjunction with modular theory ("modular nuclearity") an important concept for the classification and nonperturbative construction of models of QFT [37] [25].

After having aquainted the reader with some of the physical requirements of the LQP formulation, we now pass to a brief description of its main mathematical support: the Tomita-Takesaki modular operator theory. This theory has its origin in the operator-algebraic aspects of group representation algebras from which Tomita took the terminology "modular" (originally referring to properties of Haar measures). A conference in the US (Baton Rouge, 1967), which took place in the middle of the 60s and which was attended by mathematicians (Tomita, Takesaki, Kadison,..) and mathematical physicists (Haag, Hugenholz, Winnink, Borchers,..), is nowadays considered as the birth of the Tomita-Takesaki modular operator theory [38]. The participating physicists obtained important aspects of that theory through their project of formulating quantum statistical mechanics directly in the thermodynamic limit (statistical mechanics of *open systems*) [6]. In their new setting, the Kubo-Martin-Schwinger property (originally an analytic shortcut for computing Gibbs traces) assumed a new conceptual role because it can be directly formulated for open quantum systems in thermal equilibrium states. Although these ideas originated independently, this conference brought them together; there is hardly any area in which the contribution of mathematicians and physicists have been that much on par as in modular operator theory/modular localization.

One reason for this perfect match was that the area of physical application of modular theory widened beyond thermal systems and became combined with the defining foundational property of QFT which has been referred to as *causal localization*. The basic fact which leads to this new connection, the Reeh-Schlieder

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[15]The breakdown of causal completeness leads to a "poltergeist" effect where degrees of freedom apparently enter from "nowhere"; one finds them in $\mathcal{O}''$ but they were not in $\mathcal{O}$.

theorem [6], was already known at the time of the Baton-Rouge conference. The analyticity properties following from the positivity of the energy-momentum spectrum secure the cyclicity of the action of operator algebras and the Einstein causality the absence of annihilators of the vacuum in algebras $\mathcal{A}(\mathcal{O})$ for regions $\mathcal{O}'' \subset \mathbb{R}^4$. This cyclic and separating property is the "standardness property" for the applicability of the Tomita-Takesaki theory. But the importance of the relation between localization and the T-T theory was only noted a decade later by Bisognano and Wichmann [6] in the context of localization in a wedge region for which the Tomita-Takesaki theory makes contact with known geometrical/physical objects.

The general T-T theory is based on the existence of an unbounded antilinear closable involution S with a dense domain $domS$ in H which contains all states of the form $A\Omega$, in case of a standard pair $(\mathcal{A},\Omega)$ [40][41]. Whereas the cyclicity secures the existence of a dense domain, the absense of annihilators of Ω in $\mathcal{A}$ insures the uniqueness of the definition.

$$S_\mathcal{O} A\Omega = A^*\Omega, \ A \in \mathcal{A} \subset B(H), \ S = J\Delta^{\frac{1}{2}} = \Delta^{-\frac{1}{2}}J \tag{9}$$
$$J \ antiunit., \ \Delta^{it} \ mod. \ unitary, \ \sigma_t(A) = Ad\Delta^{it}A$$

The existence of a polar decomposition in terms of a antiunitary J and a positive generally unbounded operator Δ follows from the closability of S (in the following S stands for the closure). The modular unitary gives rise to a *modular automorphism* group of the localized algebra $\mathcal{A}$.

As a result of the Reeh-Schlieder theorem which, as a consequence of Einstein causality is equivalent to the standardness the local pairs $(\mathcal{A}(\mathcal{O}),\Omega)$ with Ω the vacuum state, the existence of the modular objects is secured for all local pairs. However their physical interpretation is only generically known for $\mathcal{O} = W = $ wedge regions which are Poincare transforms of the t-z wedge $W_{tz} = \left\{ z > |t| \, ; \mathbf{x} \in \mathbb{R}^2 \right\}$. In that case the modular objects are $\Delta_W^{it} = U(\Lambda_W(-\pi t))$ the unitary transformation representing the W-preserving Lorentz ("boost") subgroup and J is a reflection on the edge of the wedge which is, up to a π-rotation within the edge, equal to Jost's TCP operator. Since in a theory with a complete particle interpretation (to which the considerations of this paper are restricted, unless stated otherwise) the interacting TCP operator and its incoming (free) counterpart are related by the scattering operator S_{scat}, we obtain for the corresponding J

$$J_W = S_{scat}J_{W,in} \ \ for \ all \ W$$

This expresses a property of S_{scat} which is not covered by LSZ scattering theory but turns out to be indispensable for the constructive use of modular localization in QFT: S_{scat} *is a relative modular invariant between the interacting and the associated free (particle) wedge algebra.* This property was recently used in a proof [39] which reduces the interacting case in theories for which particles are "too close" to fields to the free case, whose proof is almost trivial (see below).

The relative modular invariance of S_{scat} is the crucial property which accounts for the analytic properties of S_{scat} , which find their most important ex-

pression in the *particle crossing property*. The connection between algebraic and analytic properties is much more subtle for on-shell (referring to the mass shell) objects as the S-matrix and formfactors, than for off-shell correlation function. Since most of these properties were not understood in the 60s, it is not surprising that the project of formulating particle physics as a quantization-free on-shell project failed. The misunderstandings about the particle crossing property in the construction of the *dual model*, which later entered string theory, are a fascinating illustration of how incomplete understanding of QFT which began in the E-J conundrum (and continued in the perturbative quantization approach without seriously affecting its content) finally ended in misunderstandings in an area were the correct understanding of modular localization and its consequences for the particle crossing property really matters, namely in on-shell constructions of models of particle theory (section 7).

This has led to a deep schism within particle theory between a minority which more or less knows what has gone wrong, and the majority, which probably cannot be reached any more because it has cutoff itself from unsolved conceptual problems of QFT.

Since it is not possible to present a self-consistent complete account of the mathematical aspects of modular localization and its physical consequences in a history-motivated setting as the present one, the aim in the rest of this section will be to raise awareness about its physical content.

It has been known for a long time that the algebraic structure associated to free fields allows a functorial interpretation in which operator subalgebras of the global algebra $B(H)$ are the functorial images of certain real subspaces of the Wigner space of one-particle wave functions (the famous so-called "second quantization"[16]), in particular the spacetime localized algebras are the images of localized real subspaces. This means that the issue of localization to some extend can be studied in the simpler form of localized subspaces of the Wigner particle representation space (unitary positive energy representations of the $\mathcal{P}$-group).

These localized subspaces can be defined in a intrinsic way [43] i.e. without quantization, only using operators from the positive energy representation U of the proper Poincare group $\mathcal{P}_+$ ($det = +1$) on the direct sum of two copies of the Wigner representation u of the connected component (proper orthochronous $\mathcal{P}_+^{\uparrow}$) on the one-particle space H_1. For simplicity of notation the transformation formulas are limited to the spinless case:

$$H_1 : (\varphi_1, \varphi_2) = \int \bar{\varphi}_1(p)\varphi_2(p)\frac{d^3p}{2p_0}, \quad \hat{\varphi}(x) = \frac{1}{(2\pi)^{\frac{3}{2}}}\int e^{ipx}\varphi(p)\frac{d^3p}{2p_0} \qquad (10)$$

$$U(g)(\varphi_1 \oplus \varphi_2) = u(g)\varphi_1 \oplus u(g)\varphi_2, \quad u(a,\Lambda)\varphi(p) = e^{ipa}u(\Lambda^{-1}p)$$

$$\Theta \equiv TCP, \quad \Theta(\varphi_1 \oplus \varphi_2) = C\varphi_2 \oplus C\varphi_1, \quad C\varphi(p) = \overline{\varphi(p)} \qquad (11)$$

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[16] Not to be confused with quantization; to quote a famous saying by Ed Nelson: "quantization is an art, but second quantization is a functor".

Any $\mathcal{P}_+$ transformation can be generated from $U(g)$ and Θ. For representations with $s > 0$ the Lorentz group acts through Wigner rotations (Wigner's "little group") on the little Hilbert space which in the massive case is the 2s+1 component representation space of rotations. The massless case leads to a 2-dimensional Euclidean "little space" whose degenerate representation (trivial "little translations") form a two-component little Hilbert space (helicity), whereas faithful representation acts in an infinite dimensional Hilbert space ("infinite spin") [42]. The Lorentz transformations as well as Θ act also (through representations of the little group) on the little Hilbert space.

It is precisely through the appearance of this little Hilbert space that the problem of causal localization of states (wave functions) cannot be simply solved by Fourier transformation and adding positive frequency contributions of particles with those of negative frequency from antiparticles. Whereas in the case of the two classes of finite little spaces (the massive and zero mass finite helicity class) of positive energy Wigner representation their "covariantization" was easily achieved in terms of group theoretic methods [44] and led to local pointlike generating wave functions and fields, this third infinite spin class posed a series obstacle. Attempt to convert its members into covariant pointlike wave functions and corresponding fields remained unsuccessful and there was no understanding of the origin of this failure[17]. Weinberg dismissed this large positive energy representation class by stating that nature does not make use out of it [44]. Since all important physical properties are connected to aspects of localization which are precisely those properties which remained poorly understood, such a dismissal seems to be premature in particular in times of dark matter.

The localization problems of the infinite spin class were finally solved [43] with the help of modular localization which for different problems was already used in [4]. In fact the main theorem in that paper states [43] that all positive energy wave functions are localizable in noncompact spacelike cones and only the first two classes are in addition localizable in double cones (the causal shadow of a 3-dim. sphere). Since the (topological) core of arbitrarily small double cones is a point and that of arbitrary narrow spacelike cones is a semiinfinite spacelike string, the remaining problem consisted in actually constructing the generating fields of these representation; this was achieved in [42]. The result can be described in terms of operator-valued distributions $\Psi(x, e)$ which depend in addition to the start x of the semiinfinite string also on the the spacelike string direction e, $e^2 = -1$. They are covariant under simultaneous transformations of x and e and fulfill Einstein causality

$$[\Psi_1(x_1, e_1), \Psi_2(x_2, e_2)]_{gr} = 0, \quad x_1 + \mathbb{R}_+ e >< x_2 + \mathbb{R}_+ e_2$$

where gr stands for graded (fermionic strings anticommute).

The modular localization of states uses the following construction. With a wedge $W = (x| \ x_3 > |x_0|)$ there comes a wedge preserving one parametric group of Lorentz-transformation $\Lambda_W(\chi = -2\pi\tau)$ where χ is the hyper-

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[17]Reference [45] is an exception in that certain aspects of the localization problem were already noted.

bolic boost parameter and Θ_W denotes the operator which implements the x_0-x_3 reflection. The latter differs from the total reflection Θ by a π rotation r_W around the x_3 axis (in the x_1-x_2 plane) and therefore acts on the wave functions as $J_W = U(r_W)\Theta$. Both transformations Λ_W and J_W commute. Since the generators of one-parametric strongly continuous unitary groups are selfadjoint operators, there exists an "analytic continuation" in terms of positive unbounded operators with dense domains. This forces the W-localized wave functions to have certain analyticity properties in the momentum space rapidity θ $(p_0, p_3) = \sqrt{m^2 + p_\perp^2}(ch\theta, sh\theta)$ which relate the analytic continuation of particle wave function to the complex conjugate of the antiparticle wave function[18] Using the notation $\Delta_W^{i\tau} \equiv U(\Lambda_W(-2\pi\tau))$, the commutation with the antiunitary J_W leads to

$$S_W = J_W \mathfrak{d}_W^{\frac{1}{2}} = \mathfrak{d}_W^{-\frac{1}{2}} J_W, \ S_W^2 \subset 1, \ acts \ on \ H_1 \oplus H_1 \qquad (12)$$
$$S_W\psi = \bar{\psi}, \ K_W \equiv \{\varphi \in domS_W; S_W\varphi = +\varphi\}, \ S_W i\varphi = -i\varphi$$
$$K_W \ "is \ standard" : K_W \cap iK_W = 0, \ K_W + iK_W \ dense \ in \ H_1 \oplus H_1$$

where $\bar{\psi}$ denotes the complex conjugate wave function[19]. The properties are straightforward consequences of the commutation between the boost and the associated reflection [43]. The transformation of a wave function into its complex conjugate is represented by an unbounded operators whose definition requires the restriction to a dense domain which connects both components through analytic continuation.

The properties in (12) result simply from the commutativity of $\Lambda_W(\chi)$ with the reflection J on the edge of the wedge; since J is anti-unitary it commutes with the unitary boost, there will be a change of sign in its action on the analytic continuation of u. Hence it has all the properties of a modular Tomita operator and it is easy to check that it acts on wedge-localized wave function by complex conjugation where in the presence of charge quantum numbers the particle wave function is mapped into its antiparticle. The K-spaces $K(\mathcal{O})$ for causally closed sub-wedge regions $\mathcal{O}$ can be obtained by intersections i.e. $\cap_{W \supset \mathcal{O}} K(W)$; this intersection may however turn out to vanish (see below) if the region is "too small".

The surprise resides in the fact that the transformation of wave functions to their complex conjugate (12, second line) defined on a certain dense subset encodes the information about two geometric objects: a one-parametric modular group leaving a wedge invariant and a reflection on that wedge into its opposite; the concrete wedge depends on the dense set of definition of the map into complex conjugate. This is certainly something which is totally incomprehensible in QM; it represents a small aspect of the incomplete understanding of

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[18] If there exists an operator creating a partice, the negative frequency part associated with the antiparticle annihilation must be related to the positive frequency part of the antiparticle creation in its hermitian adjoint.

[19] Although the action of S_W is diagonal, the definition of the J_W needs the antiparticle doubling of the Wigner space.

the foundations of QFT which passes like a red thread through QFT from its inception up to the present.

The connection with causal localization is of course a property which only appears in the physical context. The general setting of modular real subspaces is a Hilbert space which contains a real subspace $K \subset H$ which is standard in the above sense. The abstract S-operator is then defined in terms of K and iK.

The above application to the Wigner representation theory of positive energy representations[20] also includes the infinite spin representations which lead to semiinfinite string-localized wave functions i.e. there are no pointlike covariant wave function-valued distributions which generate these representations; they are genuinely string-localized (which the superstring representation of the Poincaré group is not; so beware of terminology. The application of the above mentioned second quantized functor converts the modular localized subspaces into a net of $\mathcal{O}$-indexed interaction-free subalgebras $\mathcal{A}(\mathcal{O})$. Interacting field theories can of course not be obtained in this way; as mentioned before, in this case one can start with the Wightman setting or the LQP algebraic formulation with the additional assumption that the theory has a complete scattering interpretation (its Hilbert space is a Wigner-Fock space).

But as it happens often in physics, if one arrives at a foundational property which has been derived from lesser fundamental setting, one changes the setting in such a way that the less fundamental properties are derived as consequences of the foundational principle. This means in particular that renormalized perturbation and all the other (within the setting of formal power series expansions) rigorous statement must also be reproducible in the new setting; this has been verified to a large extend.

The algebraic setting in terms of modular localization also gives rise to a physically extremely informative type of inclusion of two algebras which share the vacuum state, the so-called *modular inclusions* $(\mathcal{A} \subset \mathcal{B}, \Omega_{vac})$ where modular means that the modular group of the bigger $\Delta_{\mathcal{B}}^{it}$ compresses (or extends) the smaller algebra [26]. A modular inclusion forces the two algebras automatically to be of the monad type. The above mentioned "GPS construction of a QFT" from a finite number of monads positioned in a common Hilbert space uses this concept in an essential way. It is perhaps the most forceful illustration of the holistic nature of QFT.

There are two properties which always accompany modular localization and which are interesting in their own right

- *KMS property* from restriction of global vacuum to $\mathcal{A}(\mathcal{O})$. By ignoring the world outside $\mathcal{O}$ one gains infinitely many KMS modified commutation properties with modular Hamiltonians $\hat{K}$ associated to the $\widehat{\mathcal{O}}$ restricted vacuum.

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[20]The positive energy condition is absolutely crucial for obtaining the prerequisites (12) of modular localization.

$$\langle AB \rangle = \langle Be^{-K}A \rangle, \quad \Delta = e^{-K}, \ A, B \in \mathcal{A}(\mathcal{O}), \ infinitely \ many \ \widehat{K} \ \ for \ \widehat{\mathcal{O}} \supset \mathcal{O} \tag{13}$$

$\langle AB \rangle \neq \langle A \rangle \langle B \rangle \ \ if \ \ [A, B] = 0 \ in \ contrast \ to \ QM$

For chiral theories on the lightray there is a rigorous derivation of the localization entropy for an interval with vacuum attenuation length ε (surface fuzziness) from the well-known linear length $L \to \infty$ behavior (the "one-dimensional volume factor" L). They are related as $ln\varepsilon^{-1} \sim L \times kT$. This *inverse Unruh effect* plays an important role in the full understanding of the E-J conundrum presented in the next section.

- Higher dimensional localization entropy. A rigorous derivation for $d > 1+1$ based on the split property [6] does not yet exist. As mentioned in the previous section there are two competing ideas leading to results which are different by a logarithmic factor. One is based on the analogy with the vacuum polarization caused increase in the norm of the dimensionless partial charge (4); it is also favored by 't Hooft's "brickwall" idea [31]. The other idea [46] which contains an additional log term is favored by the idea of a "weak inverse Unruh effect"[23] which is based on a "lightlike box" (closely related to a holographic projection onto the causal boundary null-surface) in which two spacelike directions account for an area factor and the logarithmic factor for a lightlike direction in a spacetime localization of the spatial surface which leads to an analogy between the infinite volume heat bath entropy with that caused by localization[21]

$$V_{n-1} (kT)^{n-1} |_{T=T_{\mathrm{mod}}} \simeq \left(\frac{R}{\Delta R} \right)^{n-2} ln \left(\frac{R}{\Delta R} \right) \tag{14}$$

  V_{n-1} is the well known thermodynamic volume factor (made dimensionless by the kT powers) and the ΔR represents the thickness of a light sheet of a sphere of radius R and corresponds to the attenuation distance for the vacuum polarization. The logarithmic factor corresponds to the mentioned lightlike length L and its fuzzy boundary so that $V^{n-2} \times L \ ^{\sim} \ V^{n-1}$ i.e. transverse volume×lightlike L written in a dimensionless way. A dimensionless matter-dependent factor (which is expected to be identical on both sides) has been omitted.

The holographic projection onto a null-surface reduces the original symmetry but at the same time leads to a vast symmetry enlargement [46] containing the infinite Bondi-Metzner-Sachs symmetry which in turn contains a copy of the Poincaré group.

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[21] Both the large distance thermodynamic divergence and the short distance "split" divergence of localized algebras involve approximations of monads by type I_∞ factors and it is suggestive to look for a connection. For n=2 there is a rigorous derivation (see last section).

There is a rather deep reason why the terminology temperature in connection with "localization temperature" has to be handled with great care. This is because the notion of temperature as measured by a thermometer is based on the zeroth thermodynamic law, whereas the KMS property refers to the second law according to which it is impossible to gain energy from equilibrium states by running a Carnot cycle (the absolute temperature). In inertial systems those two definitions coalesce (after normalization), whereas in a accelerated systems used e.g. in the Unruh Gedankenexperiment to achieve the Rindler-wedge localization, this is not the case. A closer examination shows [47] that the conclusion about "egg-boiling" and particle radiation claimed as observed by an accelerated observer are incorrect. To the extend of validity of Einstein's equivalence principle, this also affects thermal manifestation ascribed to gravity as in case of black holes [48]. This does however not lead to changes of entropical consequences. It also does not change the fact that localization-caused "thermal" behavior leads to impure KMS states by restricting pure vacuum states to subalgebras of localized observables.

5 The E-J conundrum, Jordan's model

With the *locally restricted vacuum* representing a highly impure state with respect to *all* modular Hamiltonians $H_{mod}(\breve{\mathcal{O}})$, $\breve{\mathcal{O}} \supseteq \mathcal{O}$ on local observables $A \in \mathcal{A}(\mathcal{O}) = \mathcal{A}(\mathcal{O}'')$, a fundamental conceptual difference between QFT and QM has been identified. QM (type I_∞ factor) is the conceptual home of *quantum information theory*[22], whereas in case of localized subalgebras of QFT a direct assignment of entropy and information content to a monad, if possible at all, can only be done in a limiting sense. The present work shows that QFT started with this conceptual antagonism in the E-J conundrum but its foundational understanding only started more than half a century later and is still far from its closure.

For this reason it is more than a historical retrospection to re-analyze the E-J conundrum from a contemporary viewpoint. In a modern setting Jordan's two-dimensional photon[23] model is a chiral current model. As a two-dimensional zero mass field which solves the wave equation it can be decomposed into its two u,v lightray components

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[22] Another subject which would have taken different turn with a better appreciation of the problems in transfering notions of quantum information theory to QFT is the decades lasting conflict about the problem of "black hole information loss".

[23] This terminology was quite common in the early days of field quantization before it was understood that that in contrast to QM the physical properties depend in an essential way on the spacetime dimension. Jordan's photons and his later neutrinos (in his "neutrino theory of light" [8]) do not have properties which permits to interpret the real 4-dimensional objects as higher dimensional versions in the same sense that a chain of oscillators is independent embedding space..

$$\partial_\mu \partial^\mu \Phi(t,x) = 0, \ \Phi(t,x) = V(u) + V(v), \ u = t+x, \ v = t-x \qquad (15)$$

$$j(u) = \partial_u V(u), \ j(v) = \partial_v V(v), \ \langle j(u), j(u') \rangle \sim \frac{1}{(u - u' + i\varepsilon)^2}$$

$$T(u) =: j^2(u) :, \ T(v) =: j^2(v) :, \quad [j(u), j(v)] = 0$$

The scale dimension of the chiral current is $d(j) = 1$, whereas the energy-momentum tensor (the Wick-square of j) has $d(T) = 2$; the u and v world are completely independent and it suffices to consider the fluctuation problem for one chiral component. The logarithmic infrared divergence problems of zero dimensional chiral $d(V) = 0$ fields arise from the fact that the zero mass field V, different from what happens in higher dimensions[24], are really stringlike instead of pointlike localized. In fact the V is best pictured as a semiinfinite line integral (a string) over the current [8]; this underlines that the connection between infrared behavior and string-localized quantum matter also holds for chiral models on the lightray. It contrasts with QM where the infrared aspects are not related to the infinite extension of quantum matter but rather with the *range of forces* between particles. Exponentials of string-localized quantum fields involving integration over zero mass string localized d=1+3 vectorpotentials share with the exponentials of integrals over d=1+1 currents $exp i\alpha V$ the property that their infrared behavior requires a representation which is inequivalent to the vacuum representation of the field strength or currents; the emergence of superselection rules ("Maxwell charges") is one of the more radical consequences of string-localization.

The E-J fluctuation problem can be formulated in terms of j (charge fluctuations) or T (energy fluctuations). It is useful to recall that vacuum expectations of chiral operators are invariant under the fractionally acting 3-parametric acting Möbius group (x stands for u,v)

$$U(a)j(x)U(a)^* = j(x+a), \ U(\lambda)j(x)U(\lambda)^* = \lambda j(\lambda x) \quad dilation \qquad (16)$$

$$U(\alpha)j(x)U(\alpha)^* = \frac{1}{(-sin\pi\alpha + cos\pi\alpha)^2} j(\frac{cos\pi\alpha x + sin\alpha}{-sin\pi\alpha x + cos\pi\alpha}) \ rotation$$

The next step consists in identifying the KMS property of the locally restricted vacuum with that of a global system in a thermodynamic limit state. For evident reasons it is referred to as the *inverse Unruh effect,* i.e. finding a localization-caused thermal system which corresponds (after adjusting parameters) to a heat bath thermal system. In the strong form of an isomorphism this is only possible under special circumstances which are met in the Einstein-Jordan conundrum, but not in the actual Unruh Gedankenexperiment for which the localization region is the Rindler wedge.

Theorem 1 *([23]) The global chiral operator algebra $\mathcal{A}(\mathbb{R})$ associated with the heat bath representation at temperature $\beta = 2\pi$ is isomorphic to the vacuum*

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[24]The V are semiinfinite integrals over the pointlike $j's$, just as the stringlike vectorpotentials in QED are semi-infinite integrals over pointlike field strength [36].

representation restricted to the half-line chiral algebra such that

$$(\mathcal{A}(\mathbb{R}), \Omega_{2\pi}) \cong (\mathcal{A}(\mathbb{R}_+), \Omega_{vac}) \tag{17}$$
$$(\mathcal{A}(\mathbb{R})', \Omega_{2\pi}) \cong (\mathcal{A}(\mathbb{R}_-), \Omega_{vac})$$

The isomorphism intertwines the translations of $\mathbb{R}$ with the dilations of $\mathbb{R}_+$, such that the isomorphism extends to the local algebras:

$$(\mathcal{A}((a,b)), \Omega_{2\pi}) \cong (\mathcal{A}((e^a, e^b)), \Omega_{vac}) \tag{18}$$

This can be shown by modular theory. The proof extends prior work by Borchers and Yngvason [49]. . Let $\mathcal{A}$ denote the C^* algebra associated to the chiral current j[25]. Consider a thermal state ω at the (for convenience) Hawking temperature 2π associated with the translation on the line. Let $\mathcal{M}$ be the operator algebra obtained by the GNS representation and $\Omega_{2\pi}$ the state vector associated to ω. We denote by $\mathcal{N}$ the half-space algebra of $\mathcal{M}$ and by $\mathcal{N}'\cap\mathcal{M}$ the relative commutant of $\mathcal{N}$ *in* $\mathcal{M}$. The main point is now that one can show that the modular groups $\mathcal{M}$, $\mathcal{N}$ and $\mathcal{N}'\cap\mathcal{M}$ } generate a "hidden" positive energy representation of the Möbius group $SL(2, R)/Z_2$ where hidden means that the actions have no geometric interpretation on the thermal net. The positive energy representation acts on a hidden vacuum representation for which the thermal state is now the vacuum state Ω. The relation of the previous 3 thermal algebras to their vacuum counterpart is as follows:

$$\mathcal{N} = \mathcal{A}(1, \infty), \ \mathcal{N}'\cap\mathcal{M} = \mathcal{A}(0,1), \ \mathcal{M} = \mathcal{A}(0, \infty) \tag{19}$$
$$\mathcal{M}' = \mathcal{A}(-\infty, 0), \ \mathcal{A}(-\infty, \infty) = \mathcal{M} \vee \mathcal{M}'$$
$$\mathcal{M}(a, b) = \mathcal{A}(e^{2\pi a}, e^{2\pi b}) \tag{20}$$

Here $\mathcal{M}'$ is the "thermal shadow world" which is hidden in the standard Gibbs state formalism but makes its explicit appearance in the so called *thermo-field* setting i.e. the result of the GNS description in which Gibbs states described by density matrices or the KMS stated resulting from their thermodynamic limits are described in a vector formalism. The last line expresses that the interval algebras are exponentially related.

In the theorem we used the more explicit notation

$$\mathcal{M}(a, b) = (\mathcal{A}(a, b), \Omega_{th}) = (\mathcal{A}(e^{2\pi a}, e^{2\pi b}), \Omega_{vac})$$

Moreover we see, that there is a natural space-time structure also on the shadow world i.e. on the thermal commutant to the quasilocal algebra on which this hidden symmetry naturally acts. Expressing this observation a more vernacular way: The thermal shadow world is converted into virgin living space. In conclusion, we have encountered a rich hidden symmetry lying behind the tip of an iceberg, of which the tip was first seen by Borchers and Yngvason.

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[25] One can either obtain the bounded operator algebras from the spectral decomposition of the smeared free fields $j(f)$ or from a Weyl algebra construction.

Although we have assumed the temperature to have the Hawking value $\beta = 2\pi$, the reader convinces himself that the derivation may easily be generalized to arbitrary positive β as in the Borchers-Yngvason work. A more detailed exposition of these arguments is contained in a paper *Looking beyond the Thermal Horizon: Hidden Symmetries in Chiral Models* [23].

In this way an interval of length L (one-dimensional box) passes to the size of the split distance ε which plays the role of Heisenberg's vacuum polarization cloud $\varepsilon \sim e^{-L}$. Equating the thermodynamic $L \to \infty$ with the the limit of a fuzzy localization converging against a sharp localization on the vacuum side in $(e^{-2\pi L}, e^{2\pi L})$ for $L \to \infty$ with the fuzzynes $e^{-2\pi L} \equiv \varepsilon \to 0$, the thermodynamic limit of the thermal entropy passes to that of the localization entropy in the limit of vanishing ε

$$LkT \mid_{kT=2\pi} \simeq -ln\varepsilon \tag{21}$$

where the left hand side is proportional to the (dimensionless) heat bathe entropy and the right hand side is proportional to the localization entropy.

Although it is unlikely that a localization-caused thermal system is isomorphic to a heat bath thermal situation in higher dimensions (the *strong inverse Unruh effect*), there may exist a "weak" inverse Unruh situation in which the volume factor corresponds to a logarithmically modified dimensionless area law i.e. $(\frac{R}{\Delta R})^{n-2} ln(\frac{R}{\Delta R})$ where R is the radius of a double cone , $\frac{\Delta R}{R}$ its dimensionless fuzzy surface and the box with two transverse- and one lightlike- directions is the counterpart of the spatial box so that the volume factor V corresponds to a box where one direction is lightlike. This would be different by a logarithmic factor (14) from the area law which is suggested by the analogy to the behavior of vacuum polarization of a partial charge in the sharp localization limit (see previous section) and which also appears in the Bekenstein's work and in 't Hooft's proposal to make the derivation of the Hawking radiation consistent with Bekenstein's area law with the help of a brickwall picture [31]. The present state of computational control of the split property is not able to decide between these two possibilities for n>2.

The above isomorphism shows that Jordan's situation of quantum fluctuations, i.e. fluctuations in a small subinterval of a chiral QFT restricted to a halfline, is isomorphic to Einstein's Gedankenexperiment of thermal fluctuations in a heat bath thermodynamic limit state on a line restricted to an interval. Such a tight relation, also referred to a an *inverse Unruh effect* [23], can not be expected in higher spacetime dimension. Although the thermal aspect of a restricted vacuum in QFT is a structural consequence of causal localization, the general identification of the dimensionless modular temperature with an actual temperature of a heat bath system, or, which is equivalent, the modular "time" with the physical time is not correct; the modular Hamiltonian is does not describe the inertial time for which the local temperature defined in terms of the zeroth thermodynamic law agrees with the "Carnot temperature" of the second law [47].

The mean square energy fluctuation in a subinterval requires to compute the fluctuations of integrals over the energy density $T(u)$ and compare them

to the calculation in a thermal heat bath calculation (the Einstein side). This would go beyond our modest aim of showing that both systems are structurally (independently of the chiral model) identical.

Properties of states in QFT depend on the nature of the algebra: a monad does not have pure states nor density matrices, but only admits rather singular impure states as singular (non Gibbs) KMS states. The identification of states with vectors in a Hilbert space up to phase factors becomes highly ambiguous and physically impractical outside of QM. The state in form of a linear expectation functional on an algebra and the unique vector (always modulo a phase factor) obtained by the intrinsic GNS construction [6] leads to a vector representation, but this depends on the particular state used for the GNS construction. In QM the algebras are always of the B(H) type where this distinction between vector states and state vectors is not necessary.

6 On-shell constructions from modular setting

An important new insight into "particles & fields" comes from a new conceptual view of the *crossing property* of formfactors, every formfactor is analytically connected with the vacuum polarization formfactor

$$\langle 0 \left| B \right| p_1, ..p_n \rangle^{in} = \; ^{out}\langle -\bar{p}_{k+1}.., -\bar{p}_n \left| B \right| p_1, ..p_k \rangle^{in}_{con} \tag{22}$$
$$B \in \mathcal{B}(\mathcal{O}), \; \mathcal{O} = W, \; \bar{p} = antiparticle \; of \; p$$

The S-matrix pair-crossing follows via LSZ scattering formalism from formfactor crossing. Hence formally (without the analyticity) crossing is supported by the LSZ formalism, but without analytic continuation the crossing identity is a tautology. The physical content of (22) consists in the statement that the right hand sight is not only an object which can be expressed in terms of a time-ordered correlation function within the same model, but is even the analytic continuation of another on-shell quantity: the crossed formfactor; in this form the crossing becomes highly nontrivial.

As will be seen, the process of crossing some incoming momenta into their outgoing (backward mass-shell of antiparticle) counterpart is nothing else than the cyclic KMS commutation relation[26] with a wedge affiliated Lorentz boost generator as the KMS Hamiltonian. This changes the conceptual setting of crossing from what it was thought to be at the time of the bootstrap- and the dual model- project. Who among the dual model followers has thought at the time that the foundational crossing property is in the same conceptual boot as the Unruh [20] effect? Whereas the latter will probably remain (together with the Einstein-Jordan subvolume fluctuation idea) an Gedankenexperiment (albeit one which characterizes foundational properties of a successful theory), the consequences of particle crossing are observational accessible e.g. in the

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[26] The replacement of the thermal Gibbs representation, which for open systems (in the thermodynamic limit) ceases to make mathematical sense [50], by the Kubo-Martin-Schwinger analytic boundary formulation.

comparison of the high energy limit of a process with its crossed counterpart
[21]. As long as the physical origin of the dual model crossing was not known it
was difficult to dismiss its use in Mandelstam's S-matrix based approach. But its
identification of the meromorphic dual model function with Mellin transforms
of conformal 4-pointfunctions (section 7) has shown that it is not related to
approximations to scattering amplitudes.

For a special case (elastic scattering) Bros, Epstein and Glaser [22] derived
crossing from properties of Wightman functions within the rather involved set-
ting of functions of several analytic variables. These methods are similar to
those which Källén and Wightman used in their (later abandoned) project of
finding the analyticity domain of the 3-point function. Presumably the reason
why these methods were given up at the beginning of the 70s, was that the rela-
tion between mathematical expenditure and physical gain was too unfavorable.

The modern conceptual understanding came from the recognition that cross-
ing identify is equivalent to the modular KMS identity for wedge localization[27]
together with the representation of wedge-localized multi-particle states in terms
of "emulated" expressions in terms of *interacting* wedge-localized operators act-
ing on the vacuum state [25][19]. "Emulation" involves different algebras acting
in the same Hilbert space and sharing the same $\mathcal{P}$-representation.

To get some technicalities out of the way, let us first formulate the *KMS
relation for the case without interactions*. Let $B(A)$ be a composite of a free
field $A(x)$ i.e. either a W-smeared Wick-ordered polynomial pointlike compos-
ite or a product of W-smeared Wick-ordered smeared free fields $A(f_i)$ with
$supp f \subset W$. Consider

$$\langle B : A(f_1)..A(f_k) : \rangle \neq 0, \quad B, \ A(f) \ \prec \mathcal{A}(W) \tag{23}$$

$$\overset{KMS}{=} \langle : A(f_{k+1})..A(f_n) : \Delta \ B : A(f_1)..A(f_k) : \rangle, \ \Delta^{it} = U(L(-2\pi t))$$

$$\langle 0 |B| p_1,...p_n\rangle = \langle -\bar{p}_{k+1},..,-\bar{p}_n |B| p_1,...,p_k\rangle := \left\langle \bar{p}_{tr,k+1},..,\bar{p}_{tr,n} \left| \Delta^{\frac{1}{2}} B \right| p_1,...,p_k \right\rangle \tag{24}$$

where $p_{tr} = (mch\theta, msh\theta, -p_1, -p_2)$ stands for the p in which the transverse
component is reflected. The fields affiliated with the interaction-free operator
algebra localized on the wedge obey the thermal KMS relation[28] (second line).
By carrying out the Wick contraction and converting the free fields acting on
the vacuum into particle states, one obtains the free particle form of the crossing
relation in the last line.

Another way to arrive at the last line which is closer to the structural ar-
guments in the presence of interactions (below) goes as follows. Letting the
$: A(f_{k+1})..A(f_n) :$ in the second line act as their adjoint on the bra state, the
wave functions of the particles change to wave functions of the corresponding

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[27] It shares the connection between locality and analyticity with the old derivation, but
instead of going back to the Wightman functions, the analyticity is channeled through the
more foundational properties of modular localization.

[28] The vacuum restricted to $\mathcal{A}(\mathcal{O})$ looses its global groundstate property and becomes a
thermal state.

anti-particles. Taking these wave functions outside the bra state they continue to change into the complex conjugate anti-particle wave functions with inverted transverse components. Letting $\Delta^{\frac{1}{2}}$ of the Δ act on these wave functions (analytic continuation by $i\pi$), and using the fact that they are mass-shell projections of Fourier transforms of W-localized test functions, the $i\pi$ analytic continuation leads back to the original particle wave functions but with reflected transverse components, whereas the plain wave bra state vector remains those of anti-particle. Using the density of W-localized wave functions[29] one finally obtains the crossing identity in the last line.

Now we come to the much more subtle case with interactions; we use the notation: $\mathcal{A}(W) =$ interacting algebra, $\mathcal{A}_{in}(W) =$ free algebra, the letter B stands for an operator from (or at least affiliated with) the interacting algebra. Any W-localized field affiliated to the $\mathcal{A}_{in}(W)$-algebra which creates a state in $domS_{\mathcal{B}(W)} = domS_{\mathcal{A}_{in}(W)}$ has a bijectively related image in $\mathcal{A}(W)$ ("emulation" of free field structure in $\mathcal{A}(W)$ [19][25]) denoted by a subscript:

$$: A(f_1)...A(f_k) : \longrightarrow (: A(f_1)...A(f_k) :)_{\mathcal{A}(W)}, \ suppf \subset W, \ A(f_i) \prec \mathcal{A}_{in}(W)$$
(25)

$$: A(f_1)..A(f_k) : |0\rangle = (: A(f_1)..A(f_k) :)_{\mathcal{A}(W)} |0\rangle = \left| \hat{f}_1, ..\hat{f}_k \right\rangle_{in}, \ f \rightarrow \hat{f}$$

where $\hat{f}$ is the wave-function associated with the test-function f. Its existence and uniqueness is secured by modular theory applied to the wedge region [51]. The KMS relation from which the particle crossing is to be derived reads [52]

$$\left\langle B(A_{in}^{(1)})_{\mathcal{A}(W)}(A_{in}^{(2)})_{\mathcal{A}(W)} \right\rangle = \left\langle (A_{in}^2)_{\mathcal{A}(W)} \Delta B(A_{in}^1)_{\mathcal{A}(W)} \right\rangle$$
(26)

$$\Delta(A_{in}^{(2)})_{\mathcal{A}(W)}^* |0\rangle = \Delta^{\frac{1}{2}} J_0 A_{out}^{(2)} |0\rangle$$

All free operators have been "emulated" within the interacting algebra so that the KMS relation for the wedge-localized algebra can be implemented. At the same time the emulation permits to rewrite its content in terms of particle states whenever the emulats act on the vacuum. The only emulate which cannot be reconverted in this way is the one in the middle of the left hand side (26). In this case one has to understand how emulats act on multi-particle states. This problem has been solved for *integrable* models. It is one of the fortunate consequences of the holistic properties of causal localization in QFT that integrability in the sense of explicit analytic solvability has a direct connection to that fundamental principle of QFT.

The result is that integrability can be re-expressed in terms of a simple domain property of the emulate of a single free field (a PFG operator in the sense of [51]). Whereas emulats in general only inherit the invariance property of their domains under the wedge-preserving subgroups from the modular wedge localization, the requirement that the domain is also invariant under translations

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[29]The necessary local sqare integrability of formfactors and their analytic continutations follow by inspection.

turns out to be very restrictive [51]. In $d > 1+1$ forces the S-matrix to be trivial $S_{scat} = 1$, whereas in $d = 1+1$ it forces the nontrivial S-matrices to be given by suitable combinatorial products of elastic 2-particle S-matrices[30] so that the connected higher particle contributions vanish. Each such S-matrix has an associated QFT which is unique within the assumed translation invariant domain property of the PFG which turns out to be equivalent to the existence of a Fourier transform (= temperateness) of PFGs. Such models are susceptible to solutions in closed form and are therefore called "integrable". Hence integrable QFTs are defined to those with temperate PFGs. Their S-matrix is determeined in terms of a two-particle generally matrix-valued 2-particle scattering function in the rapidity variable which fulfills unitarity and crossing. Scattering functions can be classified and computed in terms of the bootstrap relations; they are in turn uniquely related to crossing symmetric formfactors via the bootstrap-formfactor construction program [54].

In contradistiction to classical models or QM where one needs to find a complete set of "conservation laws in involution" and where integrable systems exist in every dimension, integrability in QFT is directly related to domain properties of wedge-localized PFGs under translations [51]. properties which only admit a nontrivial solution in d=1+1. The simplicity of integrable S-matrix matrices (the absence of connected parts for n>2) keep integrable models in the proximity of interaction-free models. Therefore it is not surprising that their wedge-generators (the Zamolodchikov-Faddeev algebra generators) can be obtained (in case of absence of bound states) by *deformations* of free fields [53] instead of the more complicated emulation. The wedge generators (PFG, W-smeared emulates of free fields) turn out to be the Fourier-transforms of the Z-F algebra generators

$$(A_{in}(f))_{\mathcal{A}(W)} = \int_C f(\theta) Z^*(\theta) d\theta, \ C = \partial strip, \ p = m(ch\theta, sh\theta) \qquad (27)$$
$$strip = \{z \mid 0 < Imz < \pi\}, \quad Z(\theta) \equiv Z^*(\theta + i\pi)$$
$$Z^*(z_1) Z^*(z_2) = S(z_1 - z_2) Z^*(z_2) Z^*(z_1), \ z \in C$$

Since integrable models preserve the particle number in scattering processes, the n-fold application of the creation parts $Z^*(\theta)$ to the vacuum are n-particle states. Identifying the velocity-ordered particle state with the incoming states

$$Z^*(\theta_1) Z^*(\theta_2) .. Z^*(\theta_n) |0\rangle = |\theta_1, \theta_2, ., \theta_n\rangle_{in}, \ \theta_1 > \theta_2 > .. > \theta_n \qquad (28)$$
$$a.c.(\theta_1 \leftrightarrow \theta_2) \langle 0 |B| \theta_1, \theta_2, ..\theta_n\rangle_{in} = S(\theta_1 - \theta_2) \langle 0 |B| \theta_2, \theta_1, ..\theta_n\rangle_{in}$$

the old degenerate representation related to (bosonic) statistics has been "dumped" into the incoming configuration which frees the left hand side for another non-trivial representation in of the permutation group in which the transposition of

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[30] In d=1+1 the cluster factorization does not distinguish a nontrivial elestic scattering amplitude from $S_{scat} = 1$.

two neighboring $\theta's$ involves the scattering function. This nontrivial representation takes care of the *analytic exchange* of $\theta's$ inside a formfactor (second line in (28)).

The analytic change of a θ through a k-cluster of θ on its right hand side will be a product of of scattering functions which in terms of the full k+1 S-matrix corresponds to to a *grazing shot* S-matrix

$$S_{g.s.}(\theta; \theta_1, ..\theta_k) = S^k(\theta_1, ..\theta_k)^{-1} S^{k+1}(\theta, \theta_1, ..\theta_k) \tag{29}$$

Without the restriction of integrability there is no known physical interpretation of analytic changes of orderings in terms of particle formfactors. As a result of the presence of inelastic scattering thresholds they are not meromorphic in the rapidities[31] but contain cuts on the real axis which wreck the meromorphy which was needed to obtain a representation of the permutation group in terms of the scattering function.

Assuming that all singularities arise from such threshold cuts, their local square integrability secures the validity of the crossing relation. This is seen as follows. The first step is to use the extendibility of (26) from the dense set of boundary values of in $(0, i\pi)$ analytic wave funtions to locally integrable wave functions using the assumption that the singularity structure of the formfactors is given in terms of threshold cuts. The θ- ordering on the left hand side (26) can than be achieved by extending its validity to locally square integrable wave functions with finite θ-support. This leads to the validity of (22) if the two clusters are ordered according to $(\theta_1, ..\theta_k) > (\theta_{k+1}, ..\theta_n)$ where the ordering within the two clusters is arbitrary. In (22) the crossing relation has been written as one finds it in the literature in order to be able to say that this is not correct, the cluster ordering cannot be omitted.

The crossing relation is not sufficient for starting an on-shell construction project, one needs to know in addition how a PFG acts on an n-particle state. Since nontemperate PFGs have difficult domain problems it is better to study the bilinear form $Z^*(\theta)$ between two particle states. If θ is in an ordered position with respect to the other θ in the bilinear form Z^* acts like an incoming creation operator. A conjecture which generalizes the findings for the integrable case can be formulated in terms of a generalized grazing shot S-matrix $S_{g.s.}$ which in this case has matrixelements to arbitrary high particle states. This is an Ansatz for an algebraization of an analytic ordering change. The resulting formulas for bilinear forms associated with non-temperate PFGs are quite involved and their consistency has yet to be tested [25].

A managable replacement of Mandelstam's on-shell construction project for nonintegrable QFT (which includes all physical relevant models) can only be expected if it turns out that the path-dependent analytic ordering change can be encoded into a braidgroup-like structure.

The ideas about PFGs and emulations of wedge-localized particle states in terms of their creation with emulated free fields is best understood as an exten-

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[31] They are never meromorphic in the Mandelstam s, t variables, and the rapidities are only uniformization variables in case of integrable models.

sion of Wigners representation-theoretical approach for noninteracting particles to the realm of interactions. The distance between the two settings is immense, but it only reflects the subtleties in the particle-field relation which has nothing in common with the particle-wave duality of QM which already found its explanation at the time of Bohr in the use of different bur equivalent representations of QM. In contrast the particle-field relation in QFT is intrinsic, its subleties cannot be simplified by changing representations. Between Wigner's 1939 representation theory enriched with modular localization and the present attempt to extend it to interactions, the middle ground is occupied by integrable QFTs. There distance to interaction free systems has made them an interesting theoretical laboratory for which most of the important questions (but not yet all !) have already been answered. This would not have been possible without the use of concepts related to modular localization.

7 Impact of modular localization on gauge theories

It is well-known that the Hilbert space formulation for renormalizable couplings of pointlike fields is limited to spin $s < 1$. For s=1 vectorpotentials one is forced to use a Krein space formulation either in the form of Gupta-Bleuler or, in the massive case, in terms of the more ghost fields containing Becchi-Rouet-Stora-Tyutin (BRST) setting. The description in the massive case starts from the observation that by adding an indefinite metric scalar Stückelberg ϕ field (two-point function with the opposite sign) to the $d_{sd} = 2$ Proca field one obtains a lower short distance dimension $d_{sd} = 1$ "Krein vectorpotential"

$$A_\mu^K(x) = A_\mu^P(x) + \partial_\mu\phi \tag{30}$$

The choice of the independent ϕ depends on the $d_{sd} = 1$ "gauge" description one wants to use (Feynman, Landau,..). The short-distance-improving nature of indefinite metric descriptions has been realized quite early in the history of particle physics. If ones interest is limited to the on-shell S-matrix (the adiabatic limit of the Bogoliubov generating operator-valued functional $S(g)$) one only needs to use the on-shell formulation of the BRST formalism which is a simple linear transformation involving the Krein potential and the BRST ghost fields but leaves the free matter field (a free spinor- or complex scalar field) unchanged.

Using this formalism Scharf [61] showed that the time-ordered products in terms of free fields which enter the formula for the S-matrix for massive spinor QED can be defined in such a way that S passes the cohomological descend to a unitary operator in Hilbert space. He emphasized that nowhere in his calculation it was necessary to think of renormalizability as requiring to generate the masse in terms of a spontaneous symmetry breaking from the massless vectormesons. The same construction works for scalar QED. In both cases the model is uniquely determined by the specification of its field content within the setting of BRST

renormalizability which includes the specification of charge or other internal symmetries of the matter fields[32]. In a similar vein the most general coupling of a real (neutral) field to a massive vectorpotential within the setting BRST renormalizability leads to the Higgs model but without the Higgs symmetry breaking mechanism. Whereas charge conservation restricts the complex matter field to appear together with its complex conjugate in the interaction density, there is no such protection against renormalization-produces odd terms in the neutral case. The critical distance to the Higgs mechanism received additional support in a more recent paper by other authors [62].

This is not the first time that the Higgs mechanism appeared as a quasiclassical metaphor of a more foundational principle. The main motivation which led to the Higgs mechanism was the idea that by starting from zero mass vectorpotentials and generating a mass through spontaneous symmetry breaking the renormalizability is preserved in each step and hence the resulting massive vectormeson coupling will be renormalizable, even though $d_{sc}(A_\mu^P) = 2$ i.e. a direct massive coupling leads to a violation of the power-counting limit. There are two misunderstandings in this argument. First a perturbation in the zero mass setting would amount to a kind of renormalized perturbation in two parameters, the coupling and the mass term, which is even worse than the violation of the power-counting limit for which each order is well-defined up to an inceasing number of counterterms. The second misunderstanding is the belief that the massless theory (scalar QED) is renormalizable whereas its massive version is not. Actually both pointlike couplings are renormalizable in an indefinite metric Krein space setting, but since the Proca field has no massless limit one expects that it will be more difficult to extraxt physical objects acting in a Hilbert space in the massless case. That this is indeed the case will be shown later.

The better idea is to view massive vectormesons interacting with charged or neutral matter fields in the context of the Schwinger-Swieca screening mechanism [63] which states that the identically conserved Maxwell current i.e. the divergence of the field strength $F_{\mu\nu}$ leads to charge screening i.e. the "Maxwell charge" in a model, in which massive vectormesons couple to matter fields vanishes. Although the proof of this theorem is quite involved, its explicit check in massive models is very simple. Note that this theorem is only concerned with the Maxwell charge, all other charges corresponding to different currents (baryon/lepton number) remain nontrivial. In the Higgs model (the neutral coupling) the Maxwell current is the only current and its charge is screened. Conserved currents fall into 3 different types

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[32]The group structure of couplings between vectormesons is not an input but rather a consequence of Scharf's "operator gauge principle" [61].

$$screening: \ Q = \int j_0(x)d^3x = 0, \ \partial^\mu j_\mu = 0 \qquad (31)$$

$$spont.symm. - breaking: \int j_0(x)d^3x = \infty$$

$$symmetry: \int j_0(x)d^3x = finite$$

and identically conserved Maxwell currents in theories with mass gaps belong to the first class which is markedly different from the symmetry-breaking type. It is interesting to note that Swieca always referred to the "Schwinger-Higgs" screening mechanism; for him the Higgs model was a kind of charge-screened counterpart of scalar QED. His previous profound understanding of spontaneous symmetry breaking[33] prevented him from accepting the Higgs mechanism for more what it is, namely a quasiclassical metaphor, a kind of "pons asini" for those who believed that the renormalizable pointlike coupling of a massive vectormeson to neutral matter cannot be treated directly but rather needed the round about way through two-parametric scalar QED (in which the third spin degree of freedom, which is formally necessary to for obtaining massive vectormesons, is inherited from the complex matter field via Goldstone symmetry breaking). He used this terminology also in his various publications in which better accessible 2-dimensional models played the role of a "theoretical laboratory". A prominent illustration is the charge screening in Schwinger's d=1+1 massless QED which "converts" this model into one which is fully described in terms of a massive scalar free field. This model was proposed by Schwinger when he found that it was not possible to realize his screening mechanism in spinor QED; nonrenormalizable pointlike couplings as massive QED in those days were believed to be unphysical. In accordance with the historical character of the present work, it is interesting to mention that Swieca, on the occasion of a visit of Rudolf Peierls to Brazil, he asked him about his opinion about massive gauge theories. Peierls upheld the traditional view of a unique connection between a gauge principle and zero mass photons.

While cleaning up certain loopholes in Swieca's screening proof [57], Buchholz and Fredenhagen realized, that by extending the ideas which were used in its proof, one could establish a structural theorem concerning the connection between the mass gap hypothesis in the energy-momentum spectrum and localization properties of superselection charge carrying fields in theories of local observables. The theorem [59] states that although one never needs generating fields which are "more" nonlocal than semiinfinite spacelike strings (the cores of spacelike cones of arbitrary small aperture in their LQP setting). Their conjecture that the matter fields of massive gauge theories may be string-generated in this sense begs the question whether the nonrenormalizability of the pointlike formulation may be the result of forcing a pointlike localization on a situation which in a Hilbert space setting requires stringlike localization.

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[33] His 1970 Cargese [56] lecture notes represent still the most profound and comprehensive account of spontaneaous symmetry breaking.

In the Krein space setting the indefinite metric fields are formally point-like renormalizable, but what about the physical fields? For the massive vectormesons in abelian models the answer is simply obtained by passing through differentiations to the pointlike field strength; however for the matter fields there seems to exist no proposal in the existing literature and this includes the coupling of massive vectormesons to neutral scalar matter field (the Higgs model).

In fact the rational presented for the Higgs mechanism that at the end of the day one obtains a renormalizable theory in a Hilbert space which includes the Higgs field was never checked in perturbation theory; otherwise it would have been noticed that this is simply not true. A direct treatment of the Higgs model in the same Krein space (BRST) formalism which works for massive QED was given by Scharf [61]. The BRST formalism permits to calculates the physical S-matrix but the cohomological descend from Krein- to Hilbert space did not work for the matter fields. In fact this problem was already noticed in the old (Gupta-Bleuler) renormalization treatment of massive QED [69]; there was a problem with the matter field in the "unitary gauge" which did not seem to exist as a Wightman field (operator-valued distribution) but was much more singular at short distances. Of course the problem of the physical matter content was also unsolved in massless QED but there one could subsume it under the label of insufficiently understood infrared problems. This is of course not specific for the Higgs model, it rather affects all couplings involving massive vectormesons.

The answer to all these questions is: follow the message in section 3 to achieve renormalizability not by making compromises with respect to the Hilbert space structure, but by passing from pointlike to stringlike localization. In this case the starting observation is the relation between pointlike Proca fields and stringlike vectorpotentials in Hilbert space (see section 3 for notation):

$$A_\mu(x,e) = A_\mu^P(x) + \partial_\mu\phi(x,e), \ \ d_{sd}(A_\mu) = 1, \ d_{sd}(\phi) = 1, \ d_{sd}(A_\mu^P) = 2 \quad (32)$$
$$d_e A_\mu(x,e) = \partial_\mu d_e\phi(x,e), \ d_e\phi(x,e) = exact \ one - form$$
$$\psi(x,e) = \ "e^{ig\phi(x,e)}\psi(x)"$$

The string-localized perturbation theory is based on the adiabatic equivalence principle which is a perturbative implementation of the relative locality of the pointlike Proca potential to the stringlike potential (i.e. both are members of the same Borchers class). It turns out that this requirement leads to the relation in which the string-localized fields are related to their pointlike counterparts by a kind of operator gauge transformation in terms of the Stückelberg-like field $\phi(x,e)$ (32); the quotation mark indicate a normal product (which generalizes the Wick product) which has to be determined by implementing the *adiabatic equivalence principle* order by order.

The terminology "adiabatic" refers to the requirement that the difference between the nth order pointlike and stringlike form of the interaction should have the form of a derivative term of the same increasing short distance dimensionality as that of the nonrenormalizable pointlike interaction. In this case the renormalization-preventing terms have the form of boundary terms and drop

out in the adiabatic limit. This idea attributes a certain perturbative status to pointlike fields while maintaining their power growth in momentum space with polynomial degree which keeps increasing with the perturbative order. This is only consistent with nonperturbative localizability if the nontempered growth leads to strictly localizable fields (SLF) in the sense of Jaffe [58]. SLF are operator-valued distributions which are not of the Schwartz type in that they cannot be smeared with all compact supported test functions in spacetime but only with a dense set. Jaffe has shown that besides the exponential functions, all entire Wick-ordered power series in a scalar free field are SLF; if this continues to be true for exponentials of *interacting* $d_{sd} = 1$ fields, the pointlike fields would acquire a nonperturbative mathematical status. It is however not clear if SLF's can be used as generators of localized operator algebras in the sense of LQP.

It is interesting to note that string-localized fields entered QFT long before the problem of localization of Wigner's infinite spin representation was solved. As already mentioned, they appeared in the work of Buchholz and Fredenhagen [59]. In the course of improving some points in Swieca's screening theorem [57] these authors realized that the mass gap hypothesis allows to infer much more than screened Maxwell currents in massive theories in that they place limitations on localizability of the screened matter fields. They used the LQP formulation in terms of localized algebras and proved that the superselection-charge carrying operators associated with compact localized always admit a description in terms of the *tightest noncompact causal localization* which are limits of spacelike cones with arbitrary small opening angles. In terms of stringlike generators this corresponds to e-smeared strings $\Psi(x, f) = \int \Psi(x, e) f(e) de$ where the integration extends over a small region in a $d = 2 + 1$ de Sitter space.

The new perturbative approach requires string-like localization for all renormalizable couplings which involve fields with $s > 1/2$. However not all couplings which in the new sense are "string-renormalizable" (within the powercounting limit) are physically acceptable. They should admit observable subalgebras which are pointlike generated and these pointlike observable fields should be invariants of an adiabatic equivalence relation which is the quantum substitute of the classical notion of gauge invariance. The nonobservable stringlike fields should not admit pointlike Wightman fields in their class of relative local fields. In perturbation theory such non-Wightman pointlike fields should be nonrenormalizable and even if it turns out that they exist outside of perturbation theory in the sense of SLF Jaffe fields [58], they should not lead to compactly localized nets of local operator algebras. Their unwiedy short distance properties may explain why in the pointlike Krein space setting [61] one did not succeed to define physical matter fields.

The model-independent nonperturbative theorem in [59] showed that the spacelike direction of these strings (the "directions at infinity") can be changed by a unitary operator within the charged sector. The superselection structure is, as in the case of pointlike localizability, determined in terms of compact internal groups, more explicitly they have the composition structure of group duals [6]. Later investigations found that although these strings are present

in the localization properties of correlation functions, the on-shell properties (S-matrix, formfactors) are not different from those of models with pointlike generators. This explains why the formfactors of the field strength as used by Swieca, did not reveal that charge screening requires interpolating string-localized fields.

By the 80s it became clear that generating matter fields in (massless) QED cannot be pointlike generated [64]. There are simply no physical pointlike matter fields, not even the nonrenormalizable pointlike physical fields in the perturbative treatment of massive QED do not survive in the zero mass limit, each perturbative order develops incurable infrared divergencies. In this case one cannot even change the asymptotic direction of the string-localized matter fields; they are part of the superselection structure. The infrared divergences are stronger in massless YM interactions, they even prevent the existence of unphysical pointlike matter in covariant gauges in a Krein space setting which has been erroneously interpreted as a breakdown of perturbation theory whereas it only indicates that the perturbation theory must be formulated in terms of string-localized matter fields which automatically obrained from interactions with stringlike vectorpotentials.

It is remarkable that although historically the idea of screened Maxwell-currents and that of string-localized matter originated together with the Higgs model (charge-neutral "Maxwell-matter"), the quasiclassical Higgs mechanism remained but the more physical Schwinger-Swieca sceening description was lost in the maelstrom of time [60]. an interesting problem which should be explained by historians of science.

The rigidity property at infinity of (Maxwell) charged strings is the cause of all infrared divergence problems which one encounters if one blindly applies the standard scattering formalism to QED; this problem even lead to a breakdown of Lorentz-invariance in charged sectors[34] [6]. In case of the QED strings the quest for a natural infrared cutoff which modifies the rigid infrared clouds in an intrinsic way (so that the infrared divergencies on the mass shell of charged matter disappear) has led to an ongoing conceptual renovation [71] of one of the oldest problems which entered particle theory through the Bloch-Nordsiek model.

All the previous statements are presently obtaining a perturbative support through the use of string-localized vectorpotential in the setting of the adiabatic equivalence requirement. The second order calculations confirm the S-matrix results of the Krein space approach. Different from the latter there is no problem with the physical matter fields which were still missing in the Krein setting. The matter fields are obtained in terms of operator gauge transformation in terms of the $A_\mu(x,e)$ associated intrinsic Stückelberg field $\phi(x,e)$ (32). The strongest differences with the Krein space approach show up in Y-M models. In that case the implementation of adiabatic equivalence requires a nonlinear transformation of the multi-component Proca field

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[34]Covariant on-shell formfactor as used by Swieca only exist for interactions of massive vectomesons with matter but not in QED.

$$A_\mu(x,e) = "U(g\phi(x,e))A_\mu^P(x)" + \partial_\mu\phi(x,e) \tag{33}$$

where $U(g\phi(x,e))$ denotes a "rotation" in color space in which the intrinsic Stückelberg fields (multiplied by a coupling parameter) substitute the rotation parameters and the quotation mark refers to normal product generalizations of the Wick product. In this case the nonlinear modifications do not only affect the matter field (32) but also rotates the vectorpotential. This is marked difference to [61] where the linear cohomological BRST Q-formalism is only consistent with an additive change which is implemented in terms of a bilinear dependence of Q on ghost fields.

To call (33) an operator gauge transformation may be misunderstood as a return of the quantum implementation of the classical gauge principle, but here this a consequence of the adiabatic equivalence requirement which implements the most basic modular localization principle by insuring that the stringlike vectorpotential remains in the same local equivalence class as the (formally nonrenormalizable) pointlike Proca field. The best possible result with respect to the B-F structural theorem would be if the pointlike matter fields (with their worsening short distance behavior with perturbative order) have the status of a SLF field. If one could prove in addition that the nontemperate SLF fields cannot be used to generate localized operator algebras, the match between the operator-algebra based B-F theorem and the perturbative situation would be perfect. The borderline between compact localizable operator algebras for $s < 1$ and the necessity of noncompact (spacelike cone) localized (superselection) charge-carrying algebras with compact localizable observable for $s \geq 1$ would than attain structural significance beyond perturbation theory[35].

The adiabatic equivalence of the renormalizable stringlike to a formally non-renormalizable pointlike field breaks down in the zero mass limit of the vectormeson mass. This can be seen from (32), the Proca potential diverges and hence the stringlike potential looses its pointlike partner. Whereas pointlike localization in the massive case leads to a short distance behavior which increases with the order of perturbation, infrared convergent pointlike physical matter fields simply do not even exist in perturbation theory; the impossibility of compact localization follows from the quantum Gauss law [64][6] and its realization in terms of fluxes in narrow spacelike cones whose cores are semiinfinite spacelike strings is chosen in order to uphold as much covariance as possible. Starting from the stringlike potential one may replace the nonexistent pointlike potential e.g. by the rotational covariant Coulomb potential in which case the stringlike localization of the formally scalar Stückelberg scalar also becomes spread out. In a Krein space prescription one may be able to maintain the pointlike matter description at the high price of loosing the physical relevance (the clash between Hilbert space and pointlike localization). Since it is not possible to implement the adiabatic equvalence principle, the best perturbative approach

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[35] We remind the reader that perturbation theory cannot insure the existence of a theory associated with a scalar Wick-ordered interaction density since perturbative series are known to diverge.

consists in defining the "true" string-localized generators as massless limits of the convenient renormalizability-implementing massive generators.

In zero mass Y-M couplings these infrared divergencies in the pointlike setting even extend to the correlation of unphysical pointlike matter fields, a fact which has been incorrectly interpreted as a signal of a breakdown of perturbation for long distances whereas it only indicates the breakdown of the pointlike description even in Krein space[36]. Hence results which have been supported by ad hoc models of beta functions without a perturbative derivation of correlation functions and Callan-Symanzik equations should be repeated in the new string setting. The latter may also lead to a new assessment of the unsolved confinement problem.

Last not least the new setting of massive vectormeson couplings has changed the way one looks at the Higgs model. Stripped of its metaphoric wrappings of spontaneous symmetry breaking being followed by a tranformation of the massless Goldstone mode into the third spin degree of freedom of a massive vectormeson, it belongs to the same group as massive (spinor or scalar) QED which are nonrenormalizable in a pointlike setting but become renormalizable if one starts perturbation theory from stringlike potentials, except that the scalar field is not complex (charged) but real (neutral). The insurance that the stringloke formulation does not change the physics which one had in mind when one was formulating the nonrenormalizable pointlike interaction is the validity of the adiabatic eqivalence which secures (at least in perturbation theory) that the pointlike and the stringlike generator are in the same locality class.

This raises the question why this simplest of all vectormeson interactions did not appear before Higgs. Well, actually it did appear before in form of a $A_\mu^P(x)\partial^\mu\varphi^2$ coupling (whose completion within the renormalization formalism formalism is the Higgs coupling) [65]. Its purpose was to show the existence of a "mildly nonrenormalizable" subclass which can be treated in terms of ideas of resummation of Feynman graphs. As we know nowadays this idea was not correct in this form; but the fact that it is renormalizable in a wider localization framework shows that the belief that that it belongs to a special class of "mildly nonrenormalizable" couplings in the pointlike setting was not wrong. The motivation for the Higgs mechanism, namely the search for a mechanism which makes the interaction of matter with massive vectormesons renormalizable in the pointlike sense was incorrect; interaction in which massive vectormesons participate can by no pointlike trick be converted into pointlike renormalizable interactions; they are however renormalizable in the sense of string-localization independent of whether the matter is spinorial, charged scalar or neutral scalar.

Presently the stringlike perturbation theory has only been verified for abelian interactions. For the massive nonabelian case the implementation of the adiabatic equivalence requires the perturbative verification of the relation (33) between a pointlike Proca potential and its stringlike counterpart. As a result of its nonlinear dependence on the Stückelberg potential this is more complicated;

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[36] The BRST formalism as a linear cohomological relation of a Krein- to a Hilbert space may not be appropriate and lead to contraditions with the adiabatic equivakence in case of YM selfcouplings. This point requires further investigation.

compared with the linear BRST formula (32) there are additional coupling terms between the color components of $A_\mu(x,e)$ and the intrinsic Stückelberg field $\phi(x,e)$ which mimick the coupling to an external Higgs field. This casts doubts on the correctness of the BRST scheme for nonlinear YM selfinteractions. The cohomological nature of BRST seems to be only consistent with linear (abelian) gauge transformations (quadratic field dependence of the ghost charge). In that case it was only possible to save its consistency for YM interactions by coupling the vectorpotential to outside scalar fields; this appears like a return of the Higgs particle for different reasons than the historical Higgs mechanism, namely for maintaining the consistency of the BRST setting.

In view of the experimental results at LHC which seem to comply with the scalar neutral nature of a Higgs-like multiplet, the question arises: is it possible that a stringlike scalar Stückelberg field applied to the vacuum has scalar bound state components? By definition the noninteracting ϕ has a spin=1 component. In fact stringlike scalar fields may have any integer spin particle content [42]. This suggests the possible existence of a boundstate mechanism which is considerably different from that of pointlike fields. In the latter case boundstates would be associated with composites (monomials), but in the stringlike situation they may be described by $\phi(x,e)$ itself. Hence not only are there alternative theoretical ideas which could explain the LHC findings, but these ideas are also more physical than postulating a new particle based on a consistency argument of model in an unphysical Krein space description. In any case the Higgs issue has not been closed by the LHC experiment since for the first time after a 40 year stagnation there are now viable alternatives on the theoretical side which come from the correct formulation of the renormalization problem of YM interactions.

At this point it is expedient to use Galileos method of codification in terms of a dialog between Sagredo and Simplicio although nowadays there is no inquisition (the God in the God-particle is Einstein's "Dear Lord")

Sagredo: My dear Simplicio are you seriously claiming that the Higgs mechanism is a metaphor for the coupling of real scalar fields to a massive vectorpotential? But doesn't the renormalizability of the Higgs model show that its status is different from other models in which massive vectormesons appear?

Simplicio: The renormalizability of the Higgs mechanism is a belief based on the well-known fact that QED-like couplings of massless vectorpotentials are Gupta-Bleuler renormalizable and that spontaneous symmetry breaking respects renormalizability. But in order to formulate a consistent perturbation theory one first has to do a formal resummation which generates a mass of a vectormeson. At this point the model looses its appearance of a renormalizable model and its renormalization problems are not significantly different from that of massive QED.

Sagredo: Does this mean that the Higgs mass generation is part of the metaphor ?

Simplicio: QFT in its widest sense is that QT which (different from QM) fulfills the (quantum adapted) causal localization principle. Every "pons asini" which the calculating physicists uses to find a result which does not contradict

the quantum causal localization principle is legitimate as long as he does not confuse it with an intrinsic property of the object of his interest. The principles of QFT are not compatible with ideas which distinguish between masses which objects have from the beginning (intrinsic masses) and such which are the result of an anthropomorphic idea as that of a mass-generating "God-particle" which provides masses for all matter (including itself).

Sagredo: But does this not diminish the importance which is attributed to the LHC result which is claimed to permit only one interpretation within QFT?

Simplicio: Not necessarily. This "Higgs or death to QFT" situation only arose as a result of a more than 40 year stagnation from running against the same wall in which no serious attempt concerning the foundational underpinnings of the Standard Model were undertaken. Other mechanism, as the recent proposal to apply the string-localized perturbation theory also to massive Y-M couplings, are presently being tried out. But it is certainly true that the explanation of the sophisticated and expensive LHC experiment in terms of an additional coupling to a neutral scalar particle which does not fulfill any of QFT's conceptual needs is somewhat unsatisfactory.

Sagredo: Are you implying that the hype about "Gods particle" and its role as a universal donor of mass may have been unfounded?

Simplicio: I only wanted to remind you that the reality underlying physics does not exist to entertain physicists and the broader public with sexy stories like that what happens when an antropomorphic copy of a Higgs particle enters room in which many other antropomorphic particles are present.

Sagredo: I thank you dear friend for sharing your thoughts, but it will take me a long time to digest and verify the content of what you said. In a way it appears too simple, I am accustomed to connect conceptual depth with complexity.

A paper being submitted to a journal dedicated to the history and philosophy of physics should not be overloaded with mathematical-technical details. But on the opposite side their looms the accusation of only using hollow words which I will try to avoid by at least some indications about how adiabatic equivalence works are in order.

Let us illustrate this in the simplest example of massive QED which, as a result of the d=2 Hilbert space Proca fields, is a non-renormalizable theory in the sense that it produces counterterms whose short distance dimensions increase with the order of perturbation so that the resulting fields are not tempered distribution. In the Bogoliubov-Shirkov perturbation setting the S-matrix and the fields are obtained in terms of adiabatic limits from a generating operator functional

$$S(g\mathcal{L}) \equiv \sum_n \frac{i^n}{n!} T_n(\mathcal{L}, \ldots, \mathcal{L})(g, \ldots, g) \tag{34}$$

$$\psi_g(f) := S(g\mathcal{L})^{-1} \sum_n \frac{i^n}{n!} T_{n+1}(\mathcal{L}, \ldots, \mathcal{L}, \psi)(g, \ldots, g, f)$$

Here $\mathcal{L}$ is the interaction density (often referred to as the interaction Lagrangian, but in causal perturbation theory free fields need not be solution of Euler-Lagrange equations).

In the case of massive QED [67][68] we have two $\mathcal{L}s$ (32)

$$\mathcal{L}^P(x) = j^\mu(x)A_\mu^P(x), \ \mathcal{L}^S(x,e) = j^\mu(x)A_\mu^S(x,e) \tag{35}$$

$$j^\mu(x) = \frac{i}{2} : \varphi^*(x)\partial^\mu\varphi(x) : +h.c., \ \partial_\mu \to \partial_\mu - igA_\mu$$

$$S(g\mathcal{L}^P + f\psi) \simeq S(g\mathcal{L}^S + f\psi^S)$$

$$A_\mu^P(x) = A_\mu^S(x,e) - \partial_\mu\phi(x,e), \ \psi^P(x) = e^{ig(x)\phi(x,e)}\psi^S(x,e)$$

the first defines to the nonrenormalizable pointlike Proca interaction, whereas the second is the new stringlike interaction which, as a result of $d_{sd}(A_\mu^S) = 1$, stays within the power-counting limit; both $\mathcal{L}s$ act in Hilbert space. The second line defines the zero order current for scalar massive QED which, as a result of its expected[37] more interesting quadratic dependence on the massive vectorpotential, it is chosen as an illustrative example for the new stringlike renormalization theory. The third line expresses the adiabatic equivalence i.e. the expected affiliation of both description to the same relatively local "field class"; the formal expression of this expected local connection in terms of fields is written in the last line. We now sketch how this idea is implemented up to second order in g.

The first line in

$$\mathcal{L}^S = \mathcal{L}^P + \partial_\mu V^\mu, \ V^\mu(x,e) = j^\mu(x)\phi(x,e) \tag{36}$$

$$T\mathcal{L} \ \mathcal{L}' - \partial_\mu T \ V^\mu\mathcal{L}' - \partial'_\nu T\mathcal{L} \ V^{\nu\prime} + \partial_\mu\partial'_\nu TV^\mu V^{\nu\prime} = T\mathcal{L}^P\mathcal{L}^{P\prime}$$

$$d_e(T\mathcal{L}W' - \partial_\mu TV^\mu W') = 0, \ W' = \mathcal{L}', V^{\mu\prime}$$

shows that the two interactions only differ by a surface term which does not contribute to the first order S-matrix (the adiabatic limit $g(x) \to g$). In second oder the equality of adiabatic limit would follow from the second line. Hence one must show that the various time-ordered products can be defined in such a way that the equation in the second line holds. It is easy to see that the e-independence i.e. the vanishing of the differential form in the third line is a necessary and sufficient condition for the second line in which the individual terms still depend on e and e'. According to the previous remark one only has to check the e-independence in the tree approximation and the one-loop contribution, its validity for the total Wick-ordering is trivial and the vacuum contribution is unimportant.

The tree approximation leads up to a delta function contributions to the desired result and the delta function term is precisely the term which is necessary to convert the derivative in (35) into its covariant counterpart. The

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[37]As in the pointlike case, one only needs to start with a zero order interaction; the full interaction is obtained through renormalization.

renormalization for the one loop terms in the Epstein-Glaser setting is an extension problem of distributions. For $x \neq x'$ the validity of the third line (36) is easily verified. The extension is somewhat tricky since the so-called "central extension" in the Epstein-Glaser setting is still e-dependent; but an e-dependent finite counterterm establishes the validity of the third line (36); the remaining freedom of counterterms is of the standard e-independent type. This proofs the adiabatic extension of the second order S-matrix; the extension to the matter dependent correlation functions does not require to address any new conceptual problem.

The high consistency is also of great personal satisfaction; since the old attempts at renormalized perturbation theory [69] there was an ununderstood problem with the unitary gauge. It is also pleasing to see that the at that time rather aimless game, with non-tempered models of QFT in the early 60s [65] and its subsequent refinement Jaffe [58] which later became incorporated into the theory of hyperfunctions, has found its application in massive abelian gauge theories. If one reads carefully between the lines most of the articles on BRST have viewed this method as a transitory device. The concluding remarks in [66] already read like an anticipation of a more physical localization-based setting.

The derivation of a Callan-Symanzik equation and the computation of the mass-independent beta function with the opposite sign would be the first calculation which does not rely on the dubious idea that the long distance behavior is nonperturbative (just because the zero mass limit turned out to be infrared divergent in the pointlike description).

The proposed method to construct stringlocal correlation functions in causal perturbation theory does not solve the problem of what replaces the mass-shell restriction and what is the substitute for particles and their scattering. In QED there is no problem with photon scattering; the relevant scattering theory uses the Huygens principle and its formulation was worked out a long time ago by Buchholz [6]. He also introduces the concept of charge classes which discretizes the superselection structure which otherwise would be continuous as a result of the impossibility of changing the large distance aspects of infrared photon clouds. These ideas where recently used in order to obtain a natural infrared cutoff by using as the spacetime arena not the full Minkowski space but the (any) forward lightcone V_+ [70][71] The idea is that all massive objects eventually enter a fixed V_+ whereas the large distance effects of soft photon clouds do not. In this way the scattering in terms of partial states (states restricted to V_+) becomes a scattering of usual particles since the restriction to V_+ amounts to a natural (geometric) infrared cutoff without having to refer to a hypothetical photon resolution. Apparently the idea also works in the nonabelian case. It would be interesting to start from the stringlike perturbative formulation in Minkowski spacetime and pass to a perturbative description in V_+ which could replace the present (non-covariant) photon-inclusive cross-section recipe by a more natural prescription in which the equivalence classes of global states which coincide on V_+ serve to remove differences which are caused by infinite photon clouds but retain those caused by Maxwell charges.

The interest in string-localized generating fields started with the problem

to understand the localization of Wigner's third class of quantum matter; this was necessary in order to understand its physical properties. In the background there was always the philosophical idea that Einstein's "Dear Lord" does not assign an important role to form the material content of the universe with the third class just being there for mathematical completeness of positive energy representations. The characteristic aspect which sets it apart from QED or YM strings is that its strings cannot be represented as semiinfinite integrals over field strength i.e. "infinite spin strings" are irreducible; they cannot be constructed from compact pieces. This makes them ideal candidates for dark matter, an issue to which I will return in a separate publication.

8   Misunderstandings about particle crossing

The *bootstrap S-matrix approach* prior to the dual model was based on the particle crossing property, but since it soon ended in an unmanageable nonlinear mixture of unitarity, Poincare invariance and some vague idea about crossing, it did not reveal anything about the conceptual origin of crossing, let alone its precise formulation. Some years later the solvable d=1+1 integrable models showed that crossing was the result of a subtle analytic interplay between pole contributions and cut contribution and that it was not possible to describe particle crossing in terms of meromorphic functions in the Mandelstam s,t,u variables.

The existence of infinitely many integrable models also undermined the naive idea that general physical principles, as those on which the dream about a unique solution of the unwieldy nonlinear bootstrap project was based (a precursor of a theory of everything" except gravity), may by some magic only allow one solution (just because nobody had been able to find *any* solution of these non-linear structures). But messages coming from exactly solvable two-dimensional (integrable) showed that the uniqueness was an illusion.

The second attempt to obtain a constructive computational access to particle theory in terms of an on-shell project based on S-matrix properties was formulated by Mandelstam. In analogy to the successful use of the Jost-Lehmann-Dyson spectral representation which led to a rigorous proof of dispersion relation, Mandelstam postulated the validity of a double spectral representation for the elastic scattering amplitude as a starting point for getting access to analytic on-shell properties as the crossing property.

The area of misunderstanding of crossing started with Veneziano's [72] construction (based on properties Euler's beta function) of a meromorphic function of two variables which had an infinity of first order poles in the two variables which were related by an analytic crossing relation. The difficulties in implementing analytic crossing and the apparent uniqueness of Veneziano's construction created a lot of excitement within which a critical view had little chance. A comparison with exact particle crossing in integrable models could have revealed that there is no approximation of an S-matrix which is meromorphic in the s,t

Mandelstam variables[38]; approximations of scattering amplitudes must always retain certain aspects whereas others as unitarity may be lost; the dual model function has none, and as will be seen in a noment, there is a good reason why.

As explained in section 5, particle crossing is derived from the KMS identities for wedge-localization together with the emulation of incoming/outgoing particle states within the interacting wedge algebra; there is no relation to Veneziano's dual model crossing. In order to be totally clear and explicit on this point one needs to understand the conceptual origin of the Veneziano duality which initially appeared as the magic result of a sophisticated mathematical game.

The clarification is due to Mack, and his construction is here referred to as the "Mack-machine"; its input consists of conformal 4-point functions in arbitrary spacetime dimension, and its output are dual models i.e. meromorphic functions of three variable s,t,u with the third variable being a linear combination of the two other variables. The function is meromorphic in each variable and the meromorphic functions in the different variables are related by an analytic crossing e.g. $s \leftrightarrow t$. The construction uses conformal global operator expansions for pairs of operators, which are known to converge and applies them inside the 4-point function

$$A(x)B(y)\Omega = \sum_k \int d^4z \Delta_{A,B_.,C_k}(x,y,z)C_k(z)\Omega \tag{37}$$

$$\langle A_1(x_1)A_2(x_2)A_3(x_3)A_4(x_4)\rangle \rightarrow 3 \; different \; expansions \tag{38}$$

Each pair of operators has a converging expansion on the vacuum in which the resulting operators C_k stand for a list of composites which can be connected with the given pair through nonvanishing 3 point functions Δ. Used inside the 4-point function this leads to 3 different ways of decomposing the 4-point function into a sum over two three-point functions connected by an integrated two-point-function. Mack showed that the Mellin transform of this infinite sum over $C\prime s$ leads precisely to the pole representation of the meromorphic functions which define dual models; the position of the first order poles is given in terms of the spectrum of scale dimensions of the $C's$ which couple to the pairs. Veneziano's model corresponds to a certain chiral conformal model, but any conformal 4 point function in any spacetime dimension upon expansion of its 4-point function and Mellin transformation of the resulting series always leads to a dual model in the sense of defining a meromorphic function with first order poles which fulfills the crossing relation. What looked so magic and unique in the hands of Veneziano is "mass-produced" by the Mack-machine.

Graphically the relation is reminiscent of an identity between two types of infinite sums over Feynman graphs with particle exchanges either in Mandelstam's s or t variable. It is not surprising that in an age of particle physics in which, starting with Dirac's antiparticles in the inconsistent hole theory, many important discoveries were made in terms of a playful "make an Ansatz and correct as

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[38]The meromorphy in d=1+1 elastic scattering is in the uniformaising rapidity variable and not in s,t.

you go along" attitude[39], an interpretation in terms of particles was irresistible even though there is conceptual relation to Mandelstam's S-matrix based on-shell project. Conformal QFTs are interesting field theories from which one can learn a lot about the inner workings of the modular localization properties, but they certainly contain no information about particles and their scattering operators. Mellin transforms of 4-point functions are entirely different from scattering amplitudes; it does not make sense to apply ideas of unitarization to them as if they would define a kind of nonunitary approximation of the S-matrix. Last not least, particle poles have no conceptual relation to scale dimensions which appear in global operator expansions.

This could have been the end of a misunderstanding, and it probably would have been if not an even stranger twist would have greatly increased the mysterious aspects and with it the attractiveness of the subject. This consisted in the observation that the oscillator algebra resulting from the Fourier decomposition of a certain chiral 10-component current algebra formally related to supersymmetric version of the Polyakov action

$$\int d\sigma d\tau \sum_{\xi=\sigma,\tau} \partial_\xi X_\mu(\sigma,\tau) g^{\mu\nu} \partial^\xi X_\mu(\sigma,\tau), \ \sigma, \tau = t \pm x \qquad (39)$$

$$X = potentia\mathcal{L} \ of \ conforma\mathcal{L} \ current \ j$$

permits the representation of a positive energy representation of the Poincare group which decomposes into a discrete infinite sum of irreducible representation (an (m,s) "tower" of unlimited height).

The construction of such a tower (an infinite component field equation) from an irreducible algebraic structure was one of Majorana's project which he formulated in 1932 with the idea to achieve something similar to what the O(4,2) group representation theory does for the hydrogen atom spectrum in QM. This project was revived in the 60s where it acquired some popularity under the name "dynamic infinite group representation project" (Fronsdal, Barut, Kleinert,..[73]). The representation of the Poincare group on the irreducible oscillator algebra of the supersymmetric 10 component current algebra is the fist nontrivial solution of the Majorana project. But this is a group theoretic fact which has no relation to Mandelstam S-matrix based on-shell project.

To understand a bit better the prerequisites one need to encounter the representation of a noncompact group as a kind of internal symmetry group on a component space of a multicomponent chiral conformal algebra, it is helpful to be reminded of same basic fact of LQP in which inner symmetries arise from the (generally assumed without inner symmetries) the local net of observable algebras in the vacuum representation. The other inequivalent local representation classes (superselection sectors) can in typical cases be combined with the vacuum representation within a larger *field algebra net*. There are convincing arguments why a continuous set of superselection sectors (in the presence of

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[39]A precursor of Tegmark's later extreme "compute and shut up" maxime...

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zero mass particles as QED one must pass to charge-classes [70]) and noncompact internal symmetries of the field algebras cannot occur in higher than two dimensions. The superselection analysis is very different in d=1+1 dimensions.

As an illustration let us look at a n-component current algebra

$$\partial \Phi_k(x) = j_k(x), \ \Phi_k(x) = \int_{-\infty}^{x} j_k(x), \ \langle j_k(x) j_{\mathcal{L}}(x') \rangle \sim \delta_{k,l} \left(x - x' - i\varepsilon \right)^{-2}$$

(40)

$$Q_k = \Phi_k(\infty), \ \Psi(x, \vec{q}) = \text{''} : e^{i\vec{q}\vec{\Phi}(x)} : \text{''}, \ carries \ \vec{q} - charge$$

$$Q_k \simeq P_k, \ dim(e^{i\vec{q}\vec{\Phi}(x)}) \sim \vec{q} \cdot \vec{q} \simeq p_\mu p^\mu, \ (d_{sd}, s) \sim (m, s)$$

Here we have avoided the confusion notation X in favor of Φ for the multi-component current potential because we want to avoid a notation which may suggest the wrong idea of an operator which embeds a chiral conformal theory on a lightray (or on its compactified circle) into a n-dimensional Minkowski spacetime so that its development in time it looks like a 2-dimensional surface (a tube, in case of a chiral theory on a circle). This picture of a covariant string sweeping through a tube-like world-sheet is incorrect inasmuch as it is incorrect to think that the classical covariant particle Lagrangian $\sqrt{ds^2}$ leads to a covariant quantum embedding described in terms of a covariant operator $x_\mu^{op}(\tau)$. In fact, ignoring Lagrangian quantization, there simply exists no covariant operator whose projectors in the spectral decomposition fulfill the requirements of covariant localization, a fact which certainly was already on the mind of Wigner when he constructed relativistic particles by representation theory and not by quantization.

In the book on string theory by Polchinski he used this classical relativistic particle Lagrangian as a "trailer" for a relativistic quantum theory of a strings based on the Nambu-Goto which is described by a replacing the ds^2 under the square root by the corresponding covariant surface differential. But instead of being helpful this analogy turns out to be a squid load. Indeed the quantization of the Nambu-Goto Lagrangian according to the correct rules for quantization in the presence of a parametrization invariance resembles that of quantizing the Einstein-Hilbert action. It is certainly non-renormalizable and has no natural relation to the Poincare group which acts on the embedding Minkowski spacetime [74]. There is another approach to the square root N-G Lagrangian which is due to Pohlmeyer [75]; it is based on the observation that the classical system is integrable. So instead of confronting the problem of quantization of reparametrization invariant actions which inevitably leads to renormalization problems, he proposes to quantize the Poisson relations between the infinitely many conserved "charges". The problem with this quantization is that one looses the connection with localization in spacetime and Poincare covariance.

On the other hand the Polyakov Lagrangian has a direct relation to chiral conformal QFT, so one believes to be on conceptually safe grounds. Here the problem is that the representation of the irreducible oscillator algebra behind

the operator formalism (40) which serves for the representation of the Poincare group (and the ensuing intrinsic localization concept which comes with positive energy representation of the Poincare group [43]) is not the same as the one which localizes the chiral model on the lightray. With other words the Hilbert space representations of the oscillator algebra are different in both cases. The charge spectrum of the chiral theory is the whole $\mathbb{R}^n$ and the sigma-model fields Ψ in (40) are the charge carriers. On the other hand the spectrum of the representation of the Poincare group is positive and has gaps (mass gaps). The spectrum of the zero mode oscillator variable runs through the full spectrum of the charge superselection structure, whereas in the use of this degree of freedom for the representation theory of the Poincare group in the Majorana project its spectrum has gaps. The treacherous nature of the analogy between the mass spectrum and the conformal dimensional spectrum

$$P_\mu {}^\sim Q_\mu. \ P^{2\sim} Q^2 \tag{41}$$
$$\curvearrowright m^{2\sim} d_{scale}$$

is overlooked by string theorists. These analogies get even more seductive if one realizes that a particular discrete particle representation of the Poincare group (the superstring representation) does appears on the oscillator algebra of a 10 component supersymmetric current model (unique up to a finite discrete "M-theoretic" variation). But what has this group theoretic coincidence which represents the only known solution of the 1932 Majorana project to do with Mandelstam's on-shell S-matrix project? The answer is nothing, Majorana's projects is of a purely group theoretical kind, whereas Mandelstam aimed a dynamical particle theory which starts with the S-matrix and its analytic crossing property. In distinction to the string-localization of matter fields interacting with vectorpotentials in previous section, the representations occurring in the superstring representation are all *pointlike* generated. This was also what the calculations of the (graded) spacelike commutator of the putative string-fields by string-theorists in the 90s showed [76][77] but unfortunately this is not what they wrote, for them these points were located on a (presumably invisible) string.

The fact that the dimensional spectrum which appears in the Mellin transform of global operator expansions of two sigma-model fields in a very special chiral current model contains the spectrum of a discrete unitary representation of the Poincaré group is quite amusing, but it has nothing to do with Mandelstam's constructive on-shell project even if he himself still supports this unfortunate turn. None of the critical remarks in this section should be construed as diminishing the enormous importance of a correctly pursued constructive on-shell project for the future of particle physics. Apart from the lack of any connection between ST and an S-matrix approach, there is also no embedding of an n-component chiral current source theory into its internal symmetry target space; the localization concepts of the source theory cannot be realized simultaneously with that of the representation of the Poincare group on the target theory. We are not living in a (dimensionally reduced) target space of a chiral conformal QFT!

In fact a lower dimensional QFT can never be imbedded into a larger dimensional one, and neither is it possible to do the inverse (dimensional restriction). The Kaluza-Klein reduction can be implemented on classical Lagrangians and quasiclassical approximations of QFT, but the intrinsic modular localization structure of QFT does not allow to do this on its solution in terms of correlation functions or of nets of local algebras.

In most of the papers which were written under the influence of ST as those dealing with the Maldacena conjecture and the idea of branes inside a higher dimensional QFT, the "think as you computation moves along" attitude has led to confusions and stagnation. Often the correct concepts which could have prevented wrong conclusions existed but where lost in the maelstrom of time. One such subject is the holistic connection between causal completeness and cardinality of degrees of freedom in LQP. This and wrong conclusions which result from ignoring it will be the topic of the next section.

9 Localization and phase-space degrees of freedom

In a course on QM one learns that the number of "degrees of freedom" (quantum states) per unit cell in phase space is finite. Already in the beginning of the 60s it became clear that this not compatible with the causal localization in QFT for which the cardinality cannot be finite . The first computation revealed that the infinity is not worse than that of a compact set [78] which later became sharpened to the cardinality of a *nuclear set* [6]; together with modular localization theory it led to the important concept of modular nuclearity [6].

The physical motivation of these investigations consisted in the desire to understand the connection between field localization and the presence of particles. The ultimate aim to understand under what circumstances fields connect to particles with discrete masses and the validity of scattering theory including the important property of *asymptotic completeness*, remained an only partially achieved project up to this date. Among one of the derived results, whose importance should be seen in the context of more than 8 decades lasting attempts to verify the existence of models with interactions, is the before-mentioned recent existence proof for certain strictly renormalizable models (i.e. with realistic short distance behavior[40]) with the help of modular nuclearity.

Another important use of these ideas consists in the *exclusion* of models with properties which formally do not occur in a Lagrangian/functional quantization setting, but which one must be aware of in any attempt to formulate QFT in terms of intrinsic requirements, starting from modular localization. It is very easy to write down noninteracting models which fulfill Einstein causality but violate the causal completeness property as e.g. certain generalized free fields [35] as e.g. the conformal covariant generalized free field which results

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[40]The d=1+1 superrenormalizable theory can still be treated within a measure-theoretic functional quantization setting [32].

from a free field on a AdS spacetime through the AdS_{n+1}-CFT_n correspondence. The physical defect of such Einstein-causal fields is that they produce a "poltergeist effect" in the causal shadow region i.e. there are more degrees of freedom in $\mathcal{O}''$ than there were in the smaller spacetime region $\mathcal{O}$ of which $\mathcal{O}''$ is the causal completion. An investigation within LQP revealed that this effect is of a general nature and persists in the presence of interactions. Starting from a physical AdS theory one obtains an "overpopulated" CFT model; the correspondence preserves the cardinality of degrees of freedom, which in the lower dimensional conformal model causes an overpopulation which then leads to the "poltergeist" effect. Vice versa, the start from a physical CFT ends in an "anemic" AdS theory in which in order to encounter any degree of freedom at all one has to look at subalgebras localized in noncompact regions. The change of localization, involved in changing the spacetime dimensions of the abstract quantum matter, simply does not follow the naive picture. The latter is only supported in quasiclassical approximations or in QT without an intrinsic notion of quantum localization as QM.

A similar phenomenon happens in case one passes to a "brane" by fixing one spatial variable; as Mack showed [79] the overpopulation in a brane causes problems to distinguish spacetime- from inner- symmetries. As previously mentioned the embedding lower dimensional QFTs into higher dimensional ones and its Kaluza-Klein inverse are also not possible in QFT. Arguments based on quasiclassical approximations or manipulations with Lagrangians do not count, and an explicit argument in terms of correlation functions or nets of algebras does not exist; it would violate the holistic nature of QFT.

What is however consistent within modular localization is a *degrees of freedom reducing holographic projection* onto null-hypersurfaces (which is responsible for the area behavior of localization-entropy). It is also conceivable that the concept of compactifying a spacetime dimension while maintaining the same quantum matter (i.e. within a given model) can be achieved by converting the time into temperature by applying the rules of "thermalization" which introduce a compactification through periodicity and by afterwards converting one of the remaining noncompact spatial coordinats into time using the Euclidean-Minkowski relation which special QFTs offer. But strictly speaking, the holistic aspect of quantum matter in QFT does not support a clearcut separation between quantum matter and its appearance in spacetime; Kaluza-Klein reductions and embeddings are only possible in quasiclassical approximations to which the holistic relation between localization and degrees of freedom does not apply. QFT models were not around at the time of Kaluza and Klein.

These insights into the connection between the cardinality of degrees of freedom and localization immediately disproves the Maldacena conjecture which claims that both sides of the AdS_5-CFT_4 represent physical theories. It also delegates "brane physics" "extra dimensions", "dimensional reduction" and many other ideas which originated in the same frame of mind about particle physics as ST (shut up and compute) to the dustbin of history, except that in this case history is often still very present.

Returning again to Galileo's method of avoiding ideological attacks with the

use of the artifice of an imagined dialog, the conversation between Sagredo and Simplicio may have taken the following path:

Sagredo: Simplicio is it true what some of our friends tell me namely that you claim that the dual model and ST led to a derailment of an important part of particle theory?

Simplicio: Well, although my attitude has been critical, I have good reasons to avoid expressing my critique in this way. What prevents me is the fact that a mass shell based alternative to the quantization approach to QFT is a project in particle physics second in importance and subtlety only to the successful project of renormalized perturbation theory started by Tomonaga, Schwinger, Feynman and Dyson. Indeed, after the successful closure of the dispersion relation project it was natural to look for a "from top to bottom on-shell setting" which on the one hand is closer to scattering observables (especially in case of strong interactions), and on the other hand avoids the handicaps of perturbative series which as a consequence of their divergence do not contain informations with respect to existence problems. But it was clear, in particular to its protagonists as Stanley Mandelstam, that a foundational understanding of on-shell analytic properties of the S-matrix and formfactors was much more difficult since their relation to spectral properties and causal locality is of a more hidden kind. This applies in particular to the particle crossing relation. Saying that a project has been derailed may be misunderstood as saying that particle theory would have been better off without it.

Sagredo: Are you suggesting that this problem was too subtle for the generally extremely successful conduct of research which consisted of starting calculations built on educated guesses and correcting as the computations progress? A method which in its extreme form led Dirac to the discovery of antiparticles on the basis of an inconsistent hole theory?

Simplicio: Yes, especially when mathematical sophistication imposed on calculations is not controlled by conceptual guidance, one may arrive at mathematically consistent theories which are incorrect from a physical-conceptual viewpoint. It does not help to adopt Einstein's viewpoint about physical reality being governed by principles (Einstein's Dear Lord) and leave it to mathematics to decide what can be accepted as a principle.

Sagredo: Are you implying that this is what happened in ST and could explain why this theory, although being considered by some mathematicians as an extremely useful construct, has within by now 5 decades not led to a trustworthy physical prediction?

Simplicio: One has to be careful on this issue; there are of cause no time limits on when a theory, which is claimed to generalise our most successful QFT, has to deliver observationally verifiable results. In retrospect it is clear that the project of an S-matrix based on-shell approach was started at a time when no trustworthy knowledge about the conceptual origin of analytic and algebraic properties about on-shell properties in QFT were available beyond those which led to the dispersion relations. The dual model and ST resulted from an unhealthy mix of phenomenological beliefs with subtle mathematical observations outside any foundational conceptual guidance on the side of QFT.

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The latter could have revealed that one became the victim of a picture puzzle in which insuffiently understood aspects of chiral QFT were misred as new deep properties of on-shell particle theory.

Sagredo: Do you want to suggest that ST, quite independent of its lack of observational success, has serious conceptual flaws?

Simplicio: ST addressed problems for which one has no chance to navigate around the the insufficiently understood foundational modular localization principle since the days of the E-J conundrum; a conceptual clash was for the first time unavoidable. The precise point of impact is clear in retrospect: it was the insuffient understanding of the crossing property which is the most subtle result from the interplay between fields (the carriers of causal localization) and particles.

Sagredo: Does this mean that ST has no relation to particle theory at all ?

Simplicio: Not quite, the theorem that on the irreducible oscillator algebra of a 10 component supersymmetric abelian chiral current model one can find a positve energy representation of the Poincaré group namely the superstring representation in which the (m,s) particle spectrum is a subset of the dimensional (d,s) spectrum of the conformal model) is certainly a theorem obtained by string theorists whose veracity is not disputed by anybody. But particle physics deals with *interactions* as embodied in the S-matrix and formfactors, and this theorem contains no informations on those issues. The afore-mentioned construction of the superstring representation from a particular irreducible algebra of oscillators is a (the only known) solution of Majorana's project of an infinite component field equation. Majorana was inspirerd by the analogy with the O(4,2) hydrogen spectrum, but even the most hardened string theorist would not think that such project is relevant for our present understanding of modern particle theory. The irony is that when some people in th 60s looked for dynamic infinite component relativistic field equations in terms of extensions of the Lorentz group, they did not become aware that string theorist found an irreducible algebraic structure (the irreducible oscillator algebra associated to a 10-dimensional current algebra) which admits a representation which solves their problem (= Majorana's project).

Sagredo: But doesn't this show that at least there exists a close relation between the Moebius covariant chiral "source" representation which can be localized on the lightray, with the target representation of the 10-dimensional Poincaré group?

Simplicio: It depends what you mean by close; certainly the chiral representation and the representation on the index space of inner symmetries of the chiral model (which is probably what you mean by target representation) are representations of the same algebra, but they are not unitarily equivalent, which makes it impossible to interpret this situation as an embedding even before one gets to the geometric meaning of such a terminology.

Sagredo: ST led to many extremely popular derivatives; besides embeddings people associated with the ST community like to talk about extra dimensions, dimensional reductions, branes and holographic projections.

Simplicio: The obstacle against most of these ideas is that in QFT the index

space of charge-carrying quantum fields for $d > 1 + 1$ can only carry representations of compact groups which does not permit spacetime target embeddings. Classical field theories which indirectly inherited this concepts by reading back quantum internal symmetry (a consequence of localized representation theory [6]) into the classical setting, do not suffer from this restriction, see the covariant solutions of the Euler-Lagrange equations of $L^{\sim}\sqrt{ds^2}$. But in QFT the only "noncompact indices" are the tensor/spinor indices of what you call the source space. The reason why chiral model come close to such a situation is because they admit a continuum of superselected charges (more precisely nonrational chiral theories). In QM the "Born localization" (related to the spectral decomposition of the position operator) has no intrinsic significance; a linear oscillator chain can be pictured in any desired dimension, but this is not possible in QFT; even in Wigner's representation theory the concept of spin (little group) depends essentially on spacetime dimensions.

Wilson used his idea of an analytic continuation in spacetime dimensionality only for scalar particles (critical phenomena) and the Kaluza-Klein reduction is at most a quasiclassical idea; the relation between cardinality of phase space degrees of freedom and modular localization which is responsible for the holistic properties of QFT destroys such ideas as it also prevents the two QFTs being mathematically related by the AdS-CFT correspondence to represent physical theories on both sides; similar reasons prevent the brane projection to result in a physical model; only 't Hooft's holographic projection leads to a QFT on a null-space whose phase space density is adjusted to the lower spacetime dimensionality [46]. My dear friend Sagredo, I propose to leave this and other subjects to our future dialog.

10   Concluding remarks

Particle physics is presently in the midst of a deep crisis. After more than three decades of amazing progress starting at the end of wwII which lasted for more than 3 decades, the signs of stagnation are highly visible. Despite an ever increasing effort in manpower and number of publications over almost 5 decades, the hopes to arrive at a new foundational insight into particle theory through string theory have evaporated; a theory, which after such a long time (longer than the Maxwell-Einstein period) has not come up with an observable result and is still in doubt about what it really represents, cannot create faith in the future of particle physics.

The problem with the Standard model is different. It certainly is a successful theory, but its present formulation, in particular the quasiclassical Higgs mechanism (which is presently the widespread accepted explanation for an highly acclaimed most complex and costly LHC experimental new particle discovery) has been around since the 70s. During all these years there has never been an attempt to get from the quasiclassical description to a more foundational insight. As a result the metaphors around this mechanism have led to ideas which are incompatible with the foundational properties of QFT as e.g. the existence

of a mass creating process (the "God particle") to be distinguishable from one were particles are massive from the start.

In the present work an attempt is made to overcome this stagnation by new foundational insights which came from the solution of a problem which was around since the dawn of QFT: the E-J conundrum. The key to its solution is a recent completely intrinsic formulation (independent of field-coordinatizations) of causal localization which (as a result of its deep relation with modular operator theory) has been termed "modular localization". It explains why the restriction of the vacuum to the ensemble of spacetime-localized observables described in terms of an operator algebra $\mathcal{A}(\mathcal{O})$ is indistinguishable from the KMS equilibrium state of a statistical mechanics system, including the occurance of a "localization entropy". It became also clear why QFT, in contrast to QM, does not need Born's interpretive addition of a probability in terms of a "Gedanken-ensemble". For the reader who rubs his eyes in disbelief and wonders why this was not noticed together with renormalized perturbation theory, we argued that the letter can and has been derived by imposing covariance and finding consistent prescriptions for eliminating infinities.

Following "Murphy's law", at the first instance this incomplete understanding of causal localization had a chance to cause serious damage, it did so. This affected the first on-shell attempts as formulated by Mandelstam. On-shell analytic properties of the S-matrix and formfactors, of which the crossing property is the most prominent are, in contrast to off-shell analyticity, notoriously difficult to derive from the basic localization principles; the recent derivation of the particle crossing property with the help of modular localization [9][25] shows why in the 60s there was not much of a chance. The conduct of research based on guesswork and concistency checks failed and led to misunderstandings when it was applied to subtle properties connected to causal localization and particle crossing is perhaps the most subtle property of the particle-field connection.

The crossing property of the meromorphic function constructed by Veneziano which is the defining property of what was then called the "dual model" and led to ST, has no connection with the particle crossing property. The lack of insight into the origin of on-shell analytic properties from the foundational causal localization impeded a correction for many decades during which the misunderstanding solidified. With the recent conceptual progress around modular localization the origin of the misunderstanding was finally understood at least among some members within the small community of LQP, but this was too late and too low key to prevent a schism in particle theory; almost 5 decades of incomplete understanding of the consequences of causal localization have led to this schism and there is yet no end in sight. Fortunately the new ideas do not only reveal what went wrong, but also indicate how to reformulate the Mandelstam project which already led to a foundational understanding of integrability and a new strategy of how to go about the general case. Hopefully they will also break the schism.

Modular localization also led to a new way of looking at the problem of renormalizability by incorporating string-localized fields for $s \geq 1$, in particular stringlike vectorpotentials, in a new setting of gauge theory. In section 6 it

was shown that the idea that in this new setting the coupling of a massive vectormesons with scalar neutral fields is as renormalizable as massive QED; one does not need a quasiclassical idea as the Higgs mechanism for its renormalizability thus confirming what has been observed before in the BRST indefinite metric setting [61]. These new ideas concerning string-localized fields in Hilbert space are superior to Krein space methods which run into problems with physical matter fields. There is no need for a mass-generation for vectormesons a la Higgs, string localization renders these models renormalizable including the abelian Higgs model of a coupling of to a neutral scalar matter field (the Higgs field). There are indications that this states of affairs also prevails in YM couplings. The fact that the intrinsic Stückelberg field couples to the vectorpotential like an external Higgs field nourishes the hope that the LHC event may have a more profound explanation than that in terms of a scalar particle which only exist for the consistency of a Krein-space formulation.

The root cause of the present schism in particle theory is probably much deeper and would not simply disappear even if ST falls out of popularity. It may be more related to the way in which research is done than to the substance of the results. The most successful conduct which led to almost all important results since the inception of QFT in the middle 20s up to the formulation of the Standard Model of the 70s is that of starting a computation using the available tools and correcting if necessary as the ideas develop together with the calculation. Sometimes the new concept which emerged from a calculation was more convincing than the setting in which it was obtained. An illustration of this conduct is Dirac's discovery of the concept of antiparticle in the setting of his hole theory whose inconsistency (missing vacuum polarization processes) led to its later abandonment. This "correct as you go" way of conducting research led occasionally to incorrect results, but it was very successful up to the discovery of the Standard Model and ST in the 70s when its success began to wane.

There were always individuals who where convinced that this conduct of research in QFT, at least in the long run, cannot be sustained without a foundational support. This is evidenced by Jordan's talk at the first big international conference on particle theory 1929 in Charkov when he expressed his hope that a future QFT will not have to use the quantization "crutches" of the less fundamental classical field theory. One big step into this direction was achieved 10 years later in Wigner's famous particle classification based on positive energy representations of the Poincaré group. Even though Wigner (together with Jordan) was one of the pioneers of QFT, he always maintained a critical distance to it after his work on representation theory. His disappointment was that the Born localization, after adjusting it to the relativistic invariant inner product, did not give him the kind of intrinsic entrance into QFT he may have hoped for. The more intrinsic modular localization principle remained out of reach during his lifetime and even decades thereafter. The reason was obviously not its mathematical complexity, but rather the fact that the intuition about quantum theory of most theoreticians was formed in QM. QFT in this view was just a relativistic extension, and the idea that the quantum realization of causal localization may change the rules of the game had not yet entered.

It was Arthur Wightman and in a more radical way Rudolf Haag[41], who took up the challenge and gave the first formulations in which the umbilical quantization cord to classical theories was finally cut. The prize was a separation from the very successful observation-oriented research in QFT. Marvin Goldberger's dictum "the contribution of axiomatic QFT has been smaller than any pre-assigned epsilon" was not far off the general opinion. But during the last two decades this picture has changed. On the one hand the rapid progress obtained in the first three decades after wwII has slowed down; measured in terms of the number of publications the years since the beginning of the eighties are marked by stagnation even in the previously most innovative and physically relevant areas. For a long time the "compute, think, and correct if necessary" attitude was extremely successful for the perturbative quantization-based approach leading already during the 70s to the Standard Model as we know it now. But this attitude was not appropriate for the S-matrix-based on-shell attempts to obtain a nonperturbative entrance into particle theory. On-shell analytic properties and their algebraic formulation are simply too subtle for being understood in terms of the above traditional manner of conducting research; mathematical sophistication without physical conceptual control is no guaranty for success, as the misunderstanding of particle crossing in the dual model and ST shows.

The experimental findings at the LHC are presented as the vindication of quasiclassical metaphors about mass-generating "God particles" which already existed with lesser metaphoric ornamentations in the 70s. No reasonable theoretical alternatives to those pictures were developed which placed the experimental physicists under unhealthy pressure to decide a "Higgs or death" (of QFT) situation (the claim that massive vectormesons can only arise in this way). The realization that what has been elevated to the status of a foundational mechanism to generate masses of particles was in reality nothing more than defining renormalizable interactions of massive vectormesons with neutral scalar particles on par with massive QED may lead to a sobering disillusionment and kindle a new interest in conceptual investments into foundational unexplored aspects of QFT. This would not only be important for the future of QFT, but also help to understand some dark corners in its history as the solution of the E-J conundrum demonstrates.

In fact that process has already started (section 6). It led to corrections about the observable consequences of the Unruh effect [47], a foundational understanding of the origin and the preciseformulation of the particle crossing property which in turn gave rise to a reformulation of the S-matrix-based on-shell approach to particle theory and a radically new view [71] about "infrared problems" which started in the 30s with the famous Bloch-Nordsiek work.

Whether all these new findings are also capable to overcome the schism within particle theory remains to be seen.

Acknowledgements: Since the author is neither a historian nor a philoso-

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[41] Haag's gratitude to Wigner is expressed in the dedication of his book on local quantum physics [6].

pher of science, but rather was led to a situation in which his expertise in quantum field theory could shed some light on dark corners of its past by the late Jürgen Ehlers, my foremost but sadly posthumous thanks go to him. I also acknowledge some more recent advice from John Stachel.

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