Thermal state with quadratic interaction
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- The AQFT approach is based on the identification of a $*$-algebra $\mathcal{A}$ of physical observables.
- States are linear, positive and normalized functionals over $\mathcal{A}$;
- For free theories the Hadamard condition selects physically relevant states;
- In [Fredenhagen & Lindler 2014] a perturbative construction for thermal states for interacting theories has been proposed

$$\Omega_{\beta,V}(A) := \left.\frac{\Omega_\beta(A \star U_V(t))}{\Omega_\beta(U_V(t))}\right|_{t=i\beta} .$$

- In this talk: adiabatic limit of $\Omega_{\beta,V}$ for quadratic potential V

$$\Box\phi + m^2\phi + \lambda\phi = 0 .$$

Free theory $\Box\phi + m^2\phi = 0$

Off-shell algebra: $\mathcal{A}(M) = \mathrm{Alg}(\mathcal{P}_{\mathrm{loc}}, \star, *)$.

$\mathcal{P}_{\mathrm{loc}} :=$ local polynomial functionals $F\colon C^\infty(M) \to \mathbb{C}$.

1. $F^{(1)}[\phi] \in C^\infty_{\mathrm{c}}(M) \quad \forall\phi$;
2. $\mathrm{supp}(F) := \bigcup_\phi \mathrm{supp}(F^{(1)}[\phi])$ is compact;
3. $\mathrm{supp}\big(F^n[\phi]\big) \subseteq \{(x,\ldots,x) \in M^n\} \qquad \forall\phi$;

$$\phi \mapsto 1 \in \mathcal{P}_{\mathrm{loc}}\,, \qquad \phi \mapsto \int_M f(x)\phi^2(x)\mathrm{d}\eta(x) \in \mathcal{P}_{\mathrm{loc}}\,,$$

$$\phi \mapsto \int_{M^2} f(x)g(y)\phi(x)\phi(y)\mathrm{d}\eta(x)\mathrm{d}\eta(y) \notin \mathcal{P}_{\mathrm{loc}}\,.$$

Free theory $\Box\phi + m^2\phi = 0$

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$\star$-product and $*$-involution

1. $F^*(\phi) := \overline{F(\phi)}$;
2. $(F_1 \star F_2)(\phi) := F_1(\phi)F_2(\phi) + \sum_{n\geq 1} \frac{\hbar^n}{n!}\omega^{\otimes n}\big[F_1^{(n)}[\phi], F_2^{(n)}[\phi]\big]$.

$\omega \in C_c^\infty(M \times M)'$ Hadamard bidistribution

1. $\omega(f, \overline{f}) \geq 0$;
2. $\omega(f, h) - \omega(h, f) = iG(f, h)$, G causal propagator;
3. Microlocal spectrum condition: bound on $\mathrm{WF}(\omega)$;

Free theory $\Box\phi + m^2\phi = 0$

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$$\omega(f, g) = \int_{\mathbb{R}^3} \sum_{\pm} c_\pm(k)\widehat{f}_\pm(k)\widehat{g}_\mp(-k)\frac{\mathrm{d}k}{2\epsilon}\,, \quad \epsilon = (|k|^2 + m^2)^{1/2}\,,$$

$$c_+ + c_- \geq 0\,, \quad c_+ - c_- = 1\,, \quad c_- \in \mathcal{S}(\mathbb{R}^3)\,.$$

Free theory $\Box\phi + m^2\phi = 0$

Off-shell algebra: $\mathcal{A}(M) = \mathrm{Alg}(\mathcal{P}_{\mathrm{loc}}, \star, *)$.

- $\mathcal{A}_\omega(M) \simeq \mathcal{A}_{\omega'}(M)$ for different ω, ω';
- $\mathcal{A}_{\mathrm{on}}(M) := \mathcal{A}(M)/\mathcal{I}$ on-shell algebra;
- $\mathcal{O} \mapsto \mathcal{A}_{\mathrm{on}}(\mathcal{O})$ is an Haag-Kastler net;
- a state is a positive, linear and normalized functional $\Omega \colon \mathcal{A}(M) \to \mathbb{C}$.

$$\Omega_\omega \colon \mathcal{A}(M) \ni F \mapsto F(0) \in \mathbb{C}\,, \quad \Omega_\omega(F_f \star F_h) = \hbar\omega(f, h)\,.$$

Interacting theory

$$\Box\phi + m^2\phi + \lambda\phi^3(x) = 0\,.$$

Interacting theory

$$\Box\phi + m^2\phi + \lambda \mathbf{V_f^{(1)}[\phi](x)} = 0\,, \quad V_f \in \mathcal{P}_{\mathrm{loc}}\,, \quad \mathrm{supp}(V_f) \subseteq \mathrm{supp}(f).$$

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Time-ordered product $\cdot_T$

1. symmetric;
2. $F_1 \cdot_T F_2 = F_1 \star F_2 \quad \text{if} \quad J^{\uparrow}[\mathrm{supp}(F_1)] \cap J^{\downarrow}[\mathrm{supp}(F_2)] = \emptyset.$

- $\cdot_T$ well-defined on regular functionals $\mathcal{P}_{\mathrm{reg}} \subset \mathcal{P}_{\mathrm{loc}} \to$ extension;

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Quantum Møller operator

- $\mathsf{R}^{\hbar}_{\lambda,f}(F) := S(\lambda V_f)^{-1} \star \big(S(\lambda V_f) \cdot_T F\big)\,, \quad S(\lambda V_f) = \exp_T\left[\frac{i\lambda}{\hbar} V_f\right]$;
- $\mathcal{A}_{\lambda,f}(M) := \mathrm{Alg}(\mathsf{R}^{\hbar}_{\lambda,f}(\mathcal{P}_{\mathrm{loc}}), \star, *)) \subset \mathcal{A}(M)[[\lambda]]$;
- $\mathsf{R}^{\hbar}_{\lambda,f}(F) = F \quad \text{if} \quad V_f \gtrsim F$;
- $\mathsf{R}^{\hbar}_{\lambda,f}\left[\Box\phi + m^2\phi + \lambda V_f^{(1)}[\phi]\right] = \Box\phi + m^2\phi\,.$
- $\mathsf{R}^{\hbar}_{\lambda,f_1}(F) = U^{-1}_{f_1,f_2} \star \mathsf{R}^{\hbar}_{\lambda,f_2}(F) \star U_{f_1,f_2} \quad \text{if} \quad F \gtrsim V_{f_1-f_2}\,.$

Interacting theory

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- $\mathcal{A}_{\lambda,f}(\mathcal{O}) \simeq \mathcal{A}_{\lambda,h}(\mathcal{O})$ if $f, h \in 1_{\mathcal{O}}$ $\Rightarrow$ algebraic adiabatic limit $\mathcal{A}_\lambda(\mathcal{O})$;
- $\mathcal{O} \mapsto \mathcal{A}_{\lambda,\mathrm{on}}(\mathcal{O})$ is an Haag-Kastler net (from now on: $\mathcal{A} = \mathcal{A}(J^\uparrow\Sigma)$);
- a state is a positive, linear and normalized functional

$$\Omega_f : \mathcal{A}_{\lambda,f} \to \mathbb{C}[[\lambda]]$$

Weak adiabatic limit: $\lim_{f\to 1} \Omega_f(A)\,, \quad A \in \mathcal{A}_{\lambda,f}$.

Thermal states

Kubo-Martin-Schwinger states

$\beta \geq 0$, $\tau \in \hom(\mathbb{R}, \mathrm{Aut}(\mathcal{A}))$. $\Omega\colon \mathcal{A} \to \mathbb{C}$, (β, τ)-KMS state if

$$\Omega\big(A\tau_t B\big)\big|_{t=i\beta} = \Omega(BA)\,, \quad \forall A, B \in \mathcal{A}\,.$$

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$$V = 0\colon \quad \tau_t(F)(\phi) := F(\phi_t)\,, \quad \phi_t(s, \mathbf{x}) := \phi(s - t, \mathbf{x})\,.$$

$$\omega_\beta(f, g) = \int_{\mathbb{R}^3} \sum_{\pm} b_\pm(\beta, \epsilon) \widehat{f}_\pm(k) \widehat{g}_\mp(-k) \frac{\mathrm{d}k}{2\epsilon}\,, \quad b_\pm(\beta, \epsilon) := \frac{\mp}{e^{\mp\beta\epsilon} - 1}\,.$$

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$$V \neq 0: \quad \tau_{f,t}\left[\mathrm{R}^\hbar_{\lambda,f}(F)\right] := \mathrm{R}^\hbar_{\lambda,f}\big(\tau_t F\big)\,.$$

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$$\Omega_{\beta,\lambda,f}(A) := \frac{\Omega_\beta\big[A \star U_{\lambda,f}(t)\big]}{\Omega_\beta\big[U_{\lambda,f}(t)\big]}\Bigg|_{t=i\beta}\,.$$

[Fredenhagen & Lindner, 2014]

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$$\Omega_{\beta,\lambda,f}(A) := \Omega_\beta(A) + \sum_{n\geq 1}(-)^n \int_{\beta S_n} \Omega^c_\beta\left[A \bigotimes_{\ell=1}^{n} \tau_{iu_\ell}(K)\right]\mathrm{d}U\,.$$

[Fredenhagen & Lindner, 2014] $K = \mathrm{R}^\hbar_{\lambda,f}(\lambda\dot{V}_f)\,, \quad \dot{V}_f := V_{\partial_t f}\,.$

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$$V \neq 0: \quad \tau_{\chi h, t}\left[\mathrm{R}^\hbar_{\lambda, \chi h}(F)\right] := U_{\lambda, \chi h}(t)^{-1} \star \tau_t\left[\mathrm{R}^\hbar_{\lambda, \chi h}(F)\right] \star U_{\lambda, \chi h}(t)\,.$$

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The quadratic problem

$$Q_\mu(\phi) = \frac{1}{2}\int_M \phi^2(x) f_\mu(x) \mathrm{d}\eta(x)\,, \quad f_\mu(t,\mathbf{x}) = h(\mathbf{x})\chi_\mu(t)\,.$$

$$\chi_\mu(t) := \chi\left(\frac{t}{\mu}\right), \qquad \chi(t) = 1 \text{ for } t \geq 0, \qquad \chi(t) = 0 \text{ for } t \leq -1\,.$$

Adiabatic limit: $\lim_{\chi\to 1} \Omega_{\beta,\lambda,\chi}(A) := \lim_{\mu\to+\infty} \Omega_{\beta,\lambda,\chi_\mu}(A) =: \Omega_{\mathrm{Ad}}(A)\,.$

Theorem

The limit exists and

$$\omega_{\mathrm{Ad}}(f,g) = \int_{\mathbb{R}^3} \sum_{\pm} b_\pm(\beta,\epsilon_\lambda) \widehat{f}_{\lambda,\pm}(k) \widehat{g}_{\lambda,\mp}(-k) \frac{\mathrm{d}k}{2\epsilon_\lambda}\,,$$

$$b_\pm(\beta,\epsilon_\lambda) := \frac{\mp}{e^{\mp\beta\epsilon_\lambda} - 1}\,, \quad \epsilon_\lambda = (|k|^2 + m^2 + \lambda)^{1/2}\,, \quad \lambda > 0\,.$$

Classical Møller operator

$$\mathsf{R}^{\hbar}_{\lambda,f_\mu}(A) = (\mathsf{R}_{\lambda,f_\mu} \circ \gamma_{\lambda,f_\mu})(A)\,.$$

Classical Møller operator

- $\mathsf{R}_{\lambda,f_\mu}(F)(\phi) := (F \circ \mathsf{r}_{\lambda,Q_\mu})(\phi)\,;$
- $\mathsf{R}_{\lambda,f_\mu}(F) = F \quad \text{if} \quad Q_\mu \gtrsim F\,;$
- $\mathsf{R}_{\lambda,f_\mu}\left[\Box\phi + m^2\phi + \lambda f_\mu \phi\right] = \Box\phi + m^2\phi\,.$

$\Omega_\beta \circ \mathsf{R}_{\lambda,f_\mu}$ has 2-point function ($T^-_{k,\mu} = \overline{T^+_{k,\mu}}$)

$$\omega_{\lambda,\mu}(f,g) = \int \widetilde{f}(t,k)\widetilde{g}(t',-k)\sum_{\pm} b_\pm(\beta,\epsilon)\,T^\pm_{k,\mu}(t)\,T^\mp_{k,\mu}(t')\mathrm{d}t\mathrm{d}t'\mathrm{d}k\,.$$

$$\ddot{T}^+_{k,\mu} + \left[\epsilon^2 + (\epsilon^2_\lambda - \epsilon^2)\chi_\mu\right]T^+_{k,\mu} = 0\,, \quad T^+_{k,\mu}(t) = \frac{e^{-i\epsilon t}}{\sqrt{2\epsilon}} \quad \text{for } t \geq -1\,.$$

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D. & Gérard, 2017

$\lim_{\mu\to+\infty} \Omega_\beta \circ \mathrm{R}_{\lambda,f_\mu} = \Omega_{\mathrm{Ad,cl}}$ with

$$\omega_{\mathrm{Ad,cl}}(f,g) = \int_{\mathbb{R}^3} \sum_{\pm} b_\pm(\beta,\epsilon)\widehat{f}_{\lambda,\pm}(k)\widehat{g}_{\lambda,\mp}(-k)\frac{\mathrm{d}k}{2\epsilon_\lambda}\,.$$

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- $\gamma_{\lambda,f_\mu} : \mathcal{P}_{\mathrm{loc}}[[\lambda]] \to \mathcal{P}_{\mathrm{loc}}[[\lambda]]$
- $\gamma_{\lambda,f_\mu}[\phi(x)] = \phi(x) \,, \quad \gamma_{\lambda,f_\mu}[\phi^2(x)] = \phi^2(x) + c.$

For $F(\phi) = \int \phi(x) f(x) \mathrm{d}\eta(x) \,, \quad G(\phi) = \int \phi(x) g(x) \mathrm{d}\eta(x)$

$$\Omega^{\mathrm{c}}_{\beta}\left[\mathrm{R}^{\hbar}_{\lambda,f_\mu}(F) \star \mathrm{R}^{\hbar}_{\lambda,f_\mu}(G) \bigotimes_{\ell=1}^{n} \tau_{iu_\ell}(K)\right] = (\Omega_\beta \circ \mathrm{R}_{\lambda,f_\mu})^{\mathrm{c}}\left[FG \bigotimes_{\ell=1}^{n} \tau_{iu_\ell}(\lambda \dot{Q}_\mu)\right]$$

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$$\omega_{\beta,\lambda,\mu}(f,g) = \hbar\omega_{\lambda,\mu}(f,g) + \sum_{n\geq 1} (-)^n \int_{\beta S_n} (\Omega_\beta \circ \mathrm{R}_{\lambda,f_\mu})^{\mathrm{c}} \left[FG \bigotimes_{\ell=1}^{n} \tau_{iu_\ell}(\lambda \dot{Q}_\mu) \right] \mathrm{d}U$$

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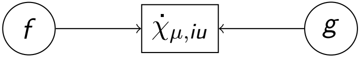
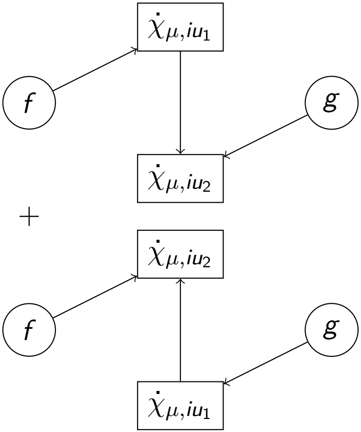
Graph expansion

$$(\Omega_\beta \circ \mathrm{R}_{\lambda, f_\mu})^{\mathrm{c}}\left[FG \otimes \tau_{iu}(\lambda \dot{Q}_\mu)\right] \sim$$



Graph expansion

$$(\Omega_\beta \circ \mathrm{R}_{\lambda,f_\mu})^{\mathrm{c}}\left[FG \bigotimes_{\ell=1}^{2} \tau_{iu_\ell}(\lambda \dot{Q}_\mu)\right] \sim$$



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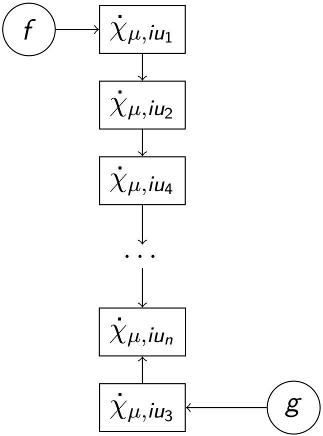

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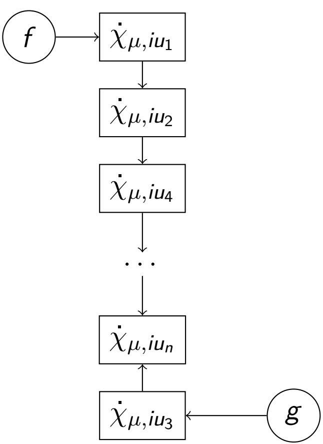
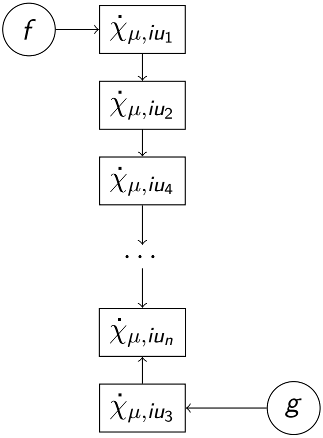


$$\mathcal{G}_n = \bigcup_{k=1}^n \mathcal{G}_{n,k} \qquad |\mathcal{G}_{n,k}| = c_{n,k}$$

$$(\Omega_\beta \circ \mathrm{R}_{\lambda,f_\mu})^{\mathrm{c}}\left[FG \bigotimes_{\ell=1}^{n} \tau_{iu_\ell}(\lambda \dot{Q}_\mu)\right] \sim \sum_{\gamma \in \mathcal{G}_n}$$



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$$\partial_\beta^n b_\pm = (-\epsilon)^n \sum_{k=1}^{n} c_{n,k} b_+^{n+1-k} b_-^k$$

$$b_\pm(\beta,\epsilon) \to b_\pm\left[\beta + \frac{\beta\lambda}{\epsilon(\epsilon+\epsilon_\lambda)}, \epsilon\right] = b_\pm(\beta, \epsilon_\lambda)$$

Conclusions

- Result expected and in agreement with [Epstein & Glaser 1973, Blanchard & Seneor 1975, Duch 2017];
- Effective resummation: $\mathsf{R}^{\hbar}_{\lambda,V} = \mathsf{R}_{\lambda,\mathbf{V}_{\mathrm{eff}}} \circ \gamma_{\lambda,V}$.
- Check for the asymptotic behaviour of the thermal two-point function. For $V \sim \phi^4$ [Bros & Buchholz 2002]

$$\Omega_{\beta,\lambda,V}\left[\mathsf{R}^{\hbar}_{\lambda,V}[\phi(x)] \star \mathsf{R}^{\hbar}_{\lambda,V}[\phi(y)]\right] \simeq \frac{\sin(\kappa(\beta)|\mathbf{x}-\mathbf{y}|)}{\kappa(\beta)|\mathbf{x}-\mathbf{y}|}\omega_\beta(x,y)\,.$$