Asymptotic Equivalence of KMS States in Rindler spacetime

Maximilian Kähler

Institute of Theoretical Physics
Leipzig University

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Introduction - The Unruh effect

"An accelerated observer perceives an ambient inertial vacuum as a state of thermal equilibrium."

[Fulling-Davies-Unruh 1973-1976]

Modern formulation in mathematical physics:

- The Minkowski vacuum restricted to the Rindler spacetime is a KMS state with real parameter[1]

$$\beta_{\mathsf{Unruh}} = \frac{2\pi}{g}.$$

Can $1/\beta_{\mathsf{Unruh}}$ be interpreted as a local temperature?

[1]natural units $c = \hbar = k_B = 1$

Outline

1 KMS Condition and Previous Results

2 Main Result

3 Proof of Main Result (Sketch)

References

D. Buchholz, C. Solveen
Unruh Effect and the Concept of Temperature
Class. Quantum Grav. 30(8):085011, Mar 2013
arXiv:1212.2409

D. Buchholz, R. Verch
Macroscopic aspects of the Unruh Effect
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KMS states

Let $(\mathcal{A}, \alpha_t)$ be a C*-algebraic dynamical system.

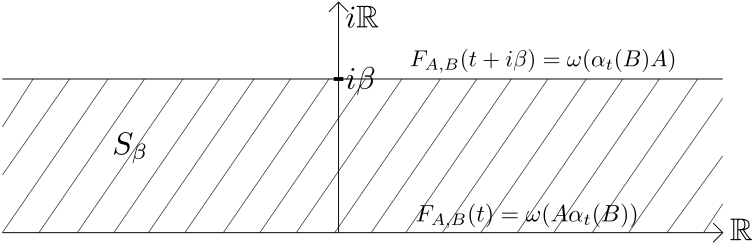
Definition

A state ω on $\mathcal{A}$ is called a β-**KMS** state for $\beta > 0$, if for all $A, B \in \mathcal{A}$ there exists a bounded continuous function

$$F_{A,B} : S_\beta := \mathbb{R} \times i[0, \beta] \longrightarrow \mathbb{C},$$

holomorphic in the interior of S_β, such that for all $t \in \mathbb{R}$

$$F_{A,B}(t) = \omega(A\alpha_t(B)), \qquad F_{A,B}(t + i\beta) = \omega(\alpha_t(B)A).$$



Recent doubts on the thermal interpretation of β_{Unruh}

Buchholz-Solveen 03/2013:

- Two distinct definitions of temperature in classical thermodynamics:
  - A) empirical temperature scale based on zeroth law,
  - B) absolute temperature scale based on second law ("Carnot Parameter").

Observation

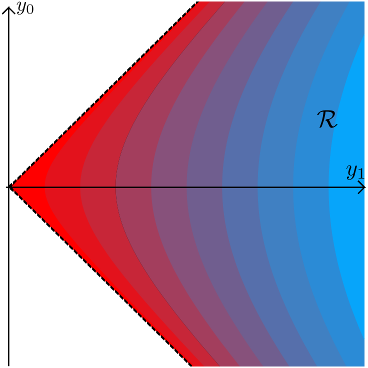
One-to-one correspondence of these definitions does only hold in inertial situations.

- Review of temperature definitions in an algebraic framework,
- KMS-parameter corresponds to the second law definition,

$\Rightarrow$ KMS-parameter loses interpretation of inverse temperature in non-inertial situations.

Buchholz and Solveen exhibit a empirical temperature observable θ_y for every $y \in \mathcal{M}$

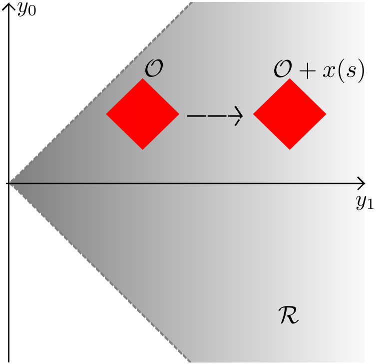
Minkowski Spacetime $\mathcal{M}$	**Rindler Spacetime $\mathcal{R}$**
$\omega_\beta^{\mathcal{M}}(\theta_y) = C\frac{1}{\beta}$	$\omega_\beta^{\mathcal{R}}(\theta_y) = \frac{C}{x(y)^2}\left(\frac{1}{\beta^2} - \frac{1}{(2\pi)^2}\right)$
spatially homogeneous	spatial temperature gradient



Far away all KMS states look the same

Buchholz-Verch 12/2014:

- Let $\mathcal{O} \subset \mathcal{R}$ be a causally complete bounded subset,
- Let $x(s) = (0, s, 0, 0) \in \mathbb{R}^4, s > 0$, be a family of 4-vectors,
- Consider the translates $\mathcal{O} + x(s)$ and the corresponding local field algebra $\mathcal{A}(\mathcal{O} + x(s))$ of a massless scalar field.



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Then for all $\beta_1, \beta_2 > 0$

$$\lim_{s \to \infty} \left\| (\omega_{\beta_1} - \omega_{\beta_2})|_{\mathcal{A}(\mathcal{O}+x(s))} \right\| = 0.$$

- spatial inhomogeneity is independent of chosen observable

Main Result

Local quasi-equivalence of quasi-free KMS states

Theorem

- *Let* $\mathcal{A}$ *be the Weyl algebra of the real massive Klein-Gordon field on the Rindler spacetime.*
- *Let* $\mathcal{A}(B)$ *be the local subalgebra corresponding to the causal completion of the unbounded region*

$$B := \left\{ (0, y_1, \boldsymbol{\xi}) \in \mathbb{R}^{1,3} \mid y_1 > X_0, \|\boldsymbol{\xi}\| < R \right\},$$

  with $R > 0, X_0 > 0$.
- *For* $\beta > 0$ *denote by* ω_β *the unique quasi-free* β*-KMS states on* $\mathcal{A}$ *with non-degenerate* β*-KMS one-particle structure.*

$\Rightarrow$ *Then for all* $0 < \beta_1 < \beta_2 \leq \infty$ *the states* $\omega_{\beta_1}|_{\mathcal{A}(B)}$ *and* $\omega_{\beta_2}|_{\mathcal{A}(B)}$ *are quasi-equivalent.*

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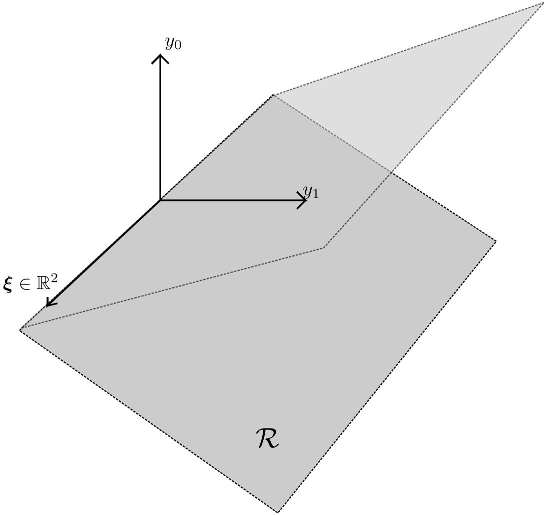
Restricted Spacetime Region in Rindler spacetime



$$\mathcal{R} := \{(y_0, y_1, y_2, y_3) \in \mathbb{R}^{1,3} \mid |y_0| < y_1\}$$

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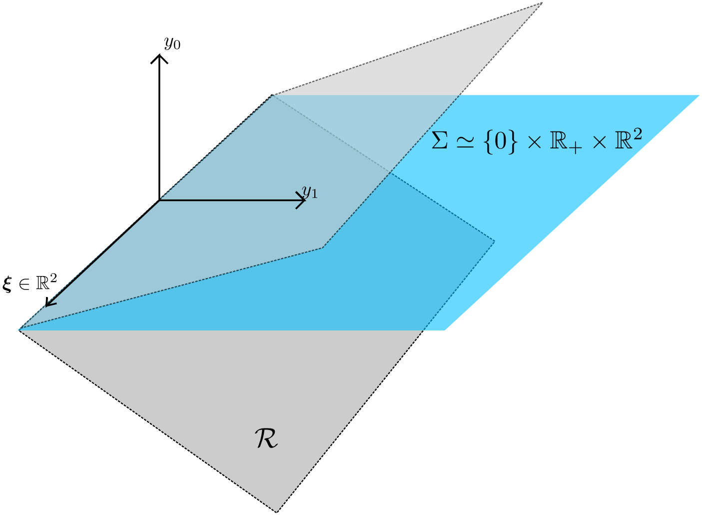

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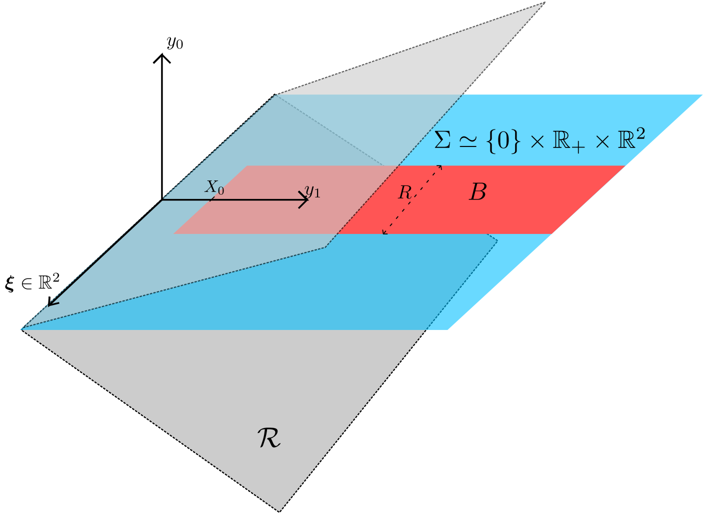

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Main Result

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$\Rightarrow$ *Then for all $0 < \beta_1 < \beta_2 \leq \infty$ the states $\omega_{\beta_1}|_{\mathcal{A}(B)}$ and $\omega_{\beta_2}|_{\mathcal{A}(B)}$ are quasi-equivalent.*

- Classical Klein-Gordon field can be described by a real symplectic space (S, σ).
- Let $\mu : S \times S \to \mathbb{R}$ be a real inner product on (S, σ), such that

$$|\sigma(\Phi_1, \Phi_2)| \leq 2\mu(\Phi_1, \Phi_1)^{1/2} \cdot \mu(\Phi_2, \Phi_2)^{1/2}.$$

  Then

$$\omega_\mu(W(\Phi)) := \exp\left(-\frac{1}{2}\mu(\Phi, \Phi)\right)$$

  defines a state on $\mathcal{A}$. ω_μ is called a **quasi-free** state.

Sufficient Criterion for Quasi-equivalence

Theorem (Araki-Yamagami 1982; Verch 1992)

Let ω_1, ω_2 *be quasi-free states on the Weyl-Algebra* $\mathcal{A}$*, uniquely characterised by real inner products* $\mu_1, \mu_2 : S \times S \to \mathbb{R}$*. Consider the complexification* $S^{\mathbb{C}} := S \oplus iS$ *and the sesquilinear extensions* $\mu_1^{\mathbb{C}}, \mu_2^{\mathbb{C}}$ *to* $S^{\mathbb{C}}$*. Then* ω_1, ω_2 *are quasi-equivalent if the following two conditions hold*

i) $\mu_1^{\mathbb{C}}, \mu_2^{\mathbb{C}}$ *induce* ***equivalent norms*** *on* $S^{\mathbb{C}}$

ii) *The operator* $T : \overline{S^{\mathbb{C}}} \to \overline{S^{\mathbb{C}}}$ *defined through*

$$\mu_1^{\mathbb{C}}(\Phi_1, \Phi_2) - \mu_2^{\mathbb{C}}(\Phi_1, \Phi_2) = \mu_1^{\mathbb{C}}(\Phi_1, T\Phi_2),$$

for all $\Phi_1, \Phi_2 \in S^{\mathbb{C}}$*, is of* ***trace class*** *in* $(\overline{S^{\mathbb{C}}}, \mu_1^{\mathbb{C}})$*.*

Defining Inner Products

- Inner product for quasi-free β-KMS states:

  $$\mu_\beta(\Phi_1, \Phi_2) = \frac{1}{2}\left(\left\langle f_1, A^{1/2}\coth\left(\frac{A^{1/2}\beta}{2}\right) f_2\right\rangle_{L^2(\mathbb{R}^3)} + \left\langle p_1, A^{-1/2}\coth\left(\frac{A^{1/2}\beta}{2}\right) p_2\right\rangle_{L^2(\mathbb{R}^3)}\right),$$

  for $\Phi_j = (f_j, p_j) \in S := C_0^\infty(\mathbb{R}^3) \times C_0^\infty(\mathbb{R}^3), j = 1, 2.$
- Involves the partial differential operator A

  $$A = -\partial_{x_1}^2 + e^{2x_1}(m^2 - \partial_{x_2}^2 - \partial_{x_3}^2),$$

  positive and essentially self-adjoint on $C_0^\infty(\mathbb{R}^3) \subset L^2(\mathbb{R}^3)$.
- Norm equivalence can be easily asserted.

Proving trace class property

- **Kontorovich-Lebedev transform:** special integral transform U provides explicit spectral representation of operator A
- **Integral operator:** U used to rewrite $T^{1/2}$ as integral operator on weighted L^2 spaces

$$\mathcal{I} := U^{-1} T^{1/2} U \colon L^2(M, \mathrm{d}\nu_{\beta_1}) \to L^2(M, \mathrm{d}\nu_{\beta_1}),$$
$$(\mathcal{I}\phi)(m) = \int_M K(m, m')\phi(m')\mathrm{d}\nu_{\beta_1}(m')$$

- **Hilbert-Schmidt Theorem:**
  $T^{1/2}$ is Hilbert-Schmidt class $\Leftrightarrow K \in L^2(M \times M, \mathrm{d}\nu_{\beta_1} \otimes \mathrm{d}\nu_{\beta_1})$

Summary

Three levels of content:

A) **Conceptual level:**

- Interpretation of Unruh effect requires careful application of thermodynamic concepts,
- $1/\beta$ need not be a meaningful temperature scale.

B) **Abstract quasi-equivalence result:**

- first result to establish local quasi-equivalence on unbounded subregion
- "Accelerated" KMS states coincide at large distance.

C) **Specific functional analytic techniques:**

- Explicit spectral calculations,
- Analysis of integral operators.

References

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Construction of the operator T

- Use the Riesz-lemma to define the operator $T : \overline{S} \to \overline{S}$ by

$$\mu_{\beta_1}(\Phi_1, \Phi_2) - \mu_{\beta_2}(\Phi_1, \Phi_2) = \mu_{\beta_1}(\Phi_1, T\Phi_2)$$

  for all $\Phi_1, \Phi_2 \in S$.
- As a matrix acting on f- and p-components of $\overline{S}$

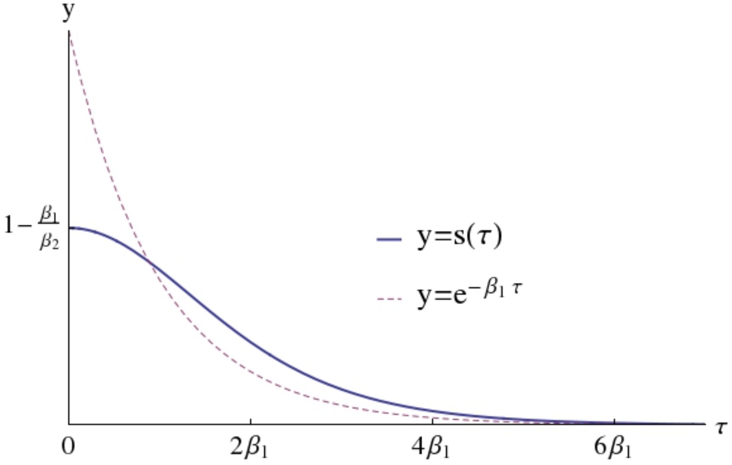
$$T = \begin{pmatrix} s_{\beta_1,\beta_2}(A^{1/2}) & 0 \\ 0 & s_{\beta_1,\beta_2}(A^{1/2}) \end{pmatrix},$$

  with

$$s_{\beta_1,\beta_2}(\tau) := \left(1 - \frac{\coth(\beta_2\tau/2)}{\coth(\beta_1\tau/2)}\right)$$

Intuition for the operator T

$$s_{\beta_1,\beta_2}(\tau) := \left(1 - \frac{\coth(\beta_2\tau/2)}{\coth(\beta_1\tau/2)}\right)$$



Previous result on local quasi-equivalence

Theorem (Verch, CMP 160)

Let ω_1 and ω_2 be two quasi-free Hadamard states on the Weyl algebra $\mathcal{A}$ of the Klein-Gordon field in some globally hyperbolic spacetime (M, g), and let π_1 and π_2 be their associated GNS representations. Then $\pi_1|_{\mathcal{A}(\mathcal{O})}$ and $\pi_2|_{\mathcal{A}(\mathcal{O})}$ are quasi-equivalent for every open subset $\mathcal{O} \subset M$ with compact closure.