Waiting for Unruh

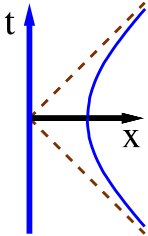
Jorma Louko

School of Mathematical Sciences, University of Nottingham

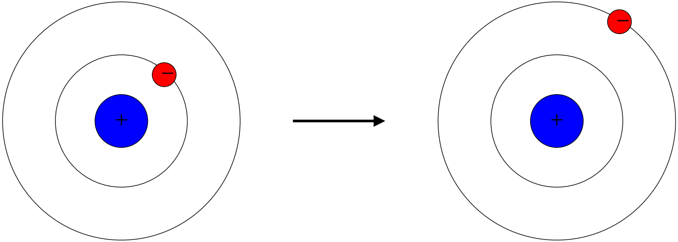
Quantum Field Theory, University of York, 4–7 April 2017

Christopher J Fewster, Benito A Juárez-Aubry, JL

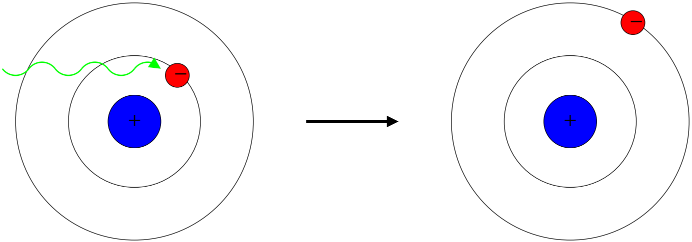
CQG 33 (2016) 165003 [arXiv:1605.01316]



Excitation



Excitation



Plan

1. **Unruh effect**
   - Long time limit: **adiabatic** scaling versus **plateau** scaling
2. **Detector**
   - Unruh-DeWitt
3. **Results**
   - Thermalisation time at large E_{gap}
4. **Summary**

1. Unruh effect

Well established

- Uniformly linearly accelerated observer sees Minkowki vacuum as thermal, $T = \frac{a}{2\pi}$ Unruh 1976
- Weak coupling, long time, negligible switching effects
- Thermal: Detector records detailed balance:

$$\boxed{\frac{P_\downarrow}{P_\uparrow} = e^{E_{\text{gap}}/T}}$$

1. Unruh effect

Well established

- Uniformly linearly accelerated observer sees Minkowki vacuum as thermal, $T = \frac{a}{2\pi}$ Unruh 1976
- Weak coupling, long time, negligible switching effects
- Thermal: Detector records detailed balance:

$$\boxed{\frac{P_{\downarrow}}{P_{\uparrow}} = e^{E_{\text{gap}}/T}}$$

Beyond: non-stationary

- Non-uniform acceleration
- Curved spacetime: Hawking effect

  E.g. detector falling into a black hole

"Time-dependent temperature" ?

Our aim

How long does a detector need to operate to record (approximate) detailed balance,

$$\boxed{\frac{P_{\downarrow}}{P_{\uparrow}} = e^{E_{\text{gap}}/T}} \quad ?$$

Our aim

How long does a detector need to operate to record (approximate) detailed balance,

$$\boxed{\frac{P_\downarrow}{P_\uparrow} = e^{E_{\text{gap}}/T}} \quad ?$$

Novel setting

- How long in terms of E_{gap}, at large E_{gap}
  $\longrightarrow$ experiment?
- Switching: smooth and compact support
- Mathematically precise (nothing hidden in $i\epsilon$)

Our aim

How long does a detector need to operate to record (approximate) detailed balance,

$$\boxed{\frac{P_{\downarrow}}{P_{\uparrow}} = e^{E_{\text{gap}}/T}} \quad ?$$

Novel setting

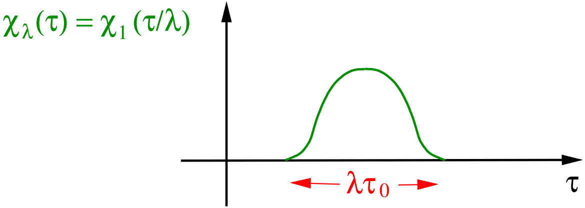
- How long in terms of E_{gap}, at large E_{gap} $\longrightarrow$ experiment?
- Switching: smooth and compact support
- Mathematically precise (nothing hidden in $i\epsilon$)

Limitations

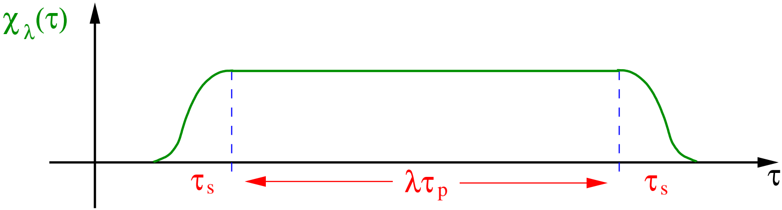
- Weak coupling $\longrightarrow$ first-order perturbation theory
- $(3+1)$ Minkowski, massless scalar field (for core results)

How long?

Adiabatic switching



Plateau switching



Long time: $\lambda \to \infty$

2. Detector

(Unruh-DeWitt)

Quantum field

$(3+1)$	spacetime dimension
ϕ	real scalar field, $m = 0$
$\|0\rangle$	Minkowski vacuum

Two-state detector (atom)

$\|\|0\rangle\rangle$	state with energy 0
$\|\|1\rangle\rangle$	state with energy E
$\mathsf{x}(\tau)$	detector worldline, τ proper time

2. Detector

(Unruh-DeWitt)

Quantum field

$(3+1)$	spacetime dimension
ϕ	real scalar field, $m=0$
$\|0\rangle$	Minkowski vacuum

Two-state detector (atom)

$\|\|0\rangle\rangle$	state with energy 0
$\|\|1\rangle\rangle$	state with energy E
$\mathsf{x}(\tau)$	detector worldline, τ proper time

Interaction

$$H_{\text{int}}(\tau) = c\chi(\tau)\mu(\tau)\phi(\mathsf{x}(\tau))$$

c	coupling constant
χ	switching function, C_0^∞, real-valued
μ	detector's monopole moment operator

Probability of transition

$$\|0\rangle\!\rangle \otimes |0\rangle \longrightarrow \|1\rangle\!\rangle \otimes |\text{anything}\rangle$$

in first-order perturbation theory:

$$P(E) = c^2 \underbrace{\left|\langle\!\langle 0\|\mu(0)\|1\rangle\!\rangle\right|^2}_{\substack{\text{detector internals only:} \\ \text{drop!}}} \times \underbrace{F(E)}_{\substack{\text{trajectory and } |0\rangle: \\ \text{response function}}}$$

$$F(E) = \int_{-\infty}^{\infty} d\tau' \int_{-\infty}^{\infty} d\tau''\, e^{-iE(\tau'-\tau'')}\, \chi(\tau')\chi(\tau'')\, W(\tau',\tau'')$$

$$W(\tau',\tau'') = \langle 0|\phi\big(x(\tau')\big)\phi\big(x(\tau'')\big)|0\rangle$$

Wightman function (distribution)

Stationary

$$W(\tau', \tau'') = W(\tau' - \tau'')$$

$$F(E) = \frac{1}{2\pi} \int_{-\infty}^{\infty} d\omega \, |\widehat{\chi}(\omega)|^2 \, \widehat{W}(E + \omega)$$

Unruh

$$\widehat{W}(\omega) = \frac{\omega}{2\pi\left(e^{2\pi\omega/a} - 1\right)}$$

$a > 0$: proper acceleration

$$\frac{\widehat{W}(-\omega)}{\widehat{W}(\omega)} = e^{2\pi\omega/a} \quad \Rightarrow \quad T = \frac{a}{2\pi}$$

Unruh temperature

3. Results

Theorem 0. With either switching, for any **fixed** E,

$$\frac{F_\lambda(E)}{\lambda} \xrightarrow[\lambda\to\infty]{} (\text{const}) \times \widehat{W}(E)$$

$\Rightarrow$ Detailed balance at $\lambda \to \infty$ (as expected)

3. Results

Theorem 0. With either switching, for any **fixed** E,

$$\frac{F_\lambda(E)}{\lambda} \xrightarrow[\lambda\to\infty]{} (\text{const}) \times \widehat{W}(E)$$

$\Rightarrow$ Detailed balance at $\lambda \to \infty$ (as expected)

Theorem 1. For fixed λ, $F_\lambda(E)$ is **not** exponentially suppressed as $E \to \infty$.

$\Rightarrow$ Detailed balance at $\lambda \to \infty$ cannot be uniform in E.

3. Results

Theorem 0. With either switching, for any **fixed** E,

$$\frac{F_\lambda(E)}{\lambda} \xrightarrow[\lambda\to\infty]{} (\text{const}) \times \widehat{W}(E)$$

$\Rightarrow$ Detailed balance at $\lambda \to \infty$ (as expected)

Theorem 1. For fixed λ, $F_\lambda(E)$ is **not** exponentially suppressed as $E \to \infty$.

$\Rightarrow$ Detailed balance at $\lambda \to \infty$ cannot be uniform in E.

Theorem 2. For **either** switching,

$$\frac{F_\lambda(-E)}{F_\lambda(E)} \xrightarrow[E\to\infty]{} e^{2\pi E/a}$$

with **exponentially** growing $\lambda(E)$

$\Rightarrow$ Detailed balance at large E_{gap} in **exponentially long waiting time**

Theorem 3. For **adiabatic** switching,

$$\frac{F_\lambda(-E)}{F_\lambda(E)} \xrightarrow[E\to\infty]{} e^{2\pi E/a} \qquad (*)$$

with **polynomially** growing $\lambda(E)$, provided $\left|\widehat{\chi}(\omega)\right|$ has sufficiently strong falloff (Cf. Fewster and Ford 2015)

$\Rightarrow$ Detailed balance at large E_{gap} in **polynomially long waiting time**

Theorem 3. For **adiabatic** switching,

$$\frac{F_\lambda(-E)}{F_\lambda(E)} \xrightarrow[E\to\infty]{} e^{2\pi E/a} \qquad (*)$$

with **polynomially** growing $\lambda(E)$, provided $|\widehat{\chi}(\omega)|$ has sufficiently strong falloff (Cf. Fewster and Ford 2015)

$\Rightarrow$ Detailed balance at large E_{gap} in **polynomially long waiting time**

Theorem 4. For **plateau** switching, no polynomially growing $\lambda(E)$ gives $(*)$

$\Rightarrow$ Detailed balance at large E_{gap} requires **longer than polynomial waiting time.**

4. Summary

Detailed balance in the Unruh effect at $E_{\text{gap}} \to \infty$:

- $(3+1)$ massless scalar
- **Polynomial** waiting time suffices for **adiabatically** scaled switching with sufficiently strong Fourier decay
- **No polynomial** waiting time suffices for **plateau** scaled switching

Upshots:

- Large E_{gap} regime has limited relevance for defining a "time dependent temperature"
- **Interest for (analogue) experiments?**