Shadows of Quantum Spacetime and pale glares of Dark Matter

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Local Quantum Physics and beyond
In memoriam Rudolf Haag

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Introduction. Why QST

QM finitely many d. o. f.

$$\Delta q \Delta p \gtrsim \hbar$$

positions = observables, dual to momentum;

in **QFT**, **local observables**:

$$A \in \mathfrak{A}(\mathcal{O});$$

$\mathcal{O}$ (*double cones*) - spacetime specifications, in terms of coordinates - accessible through measurements of local observables. **Allows to formulate LOCALITY**:

$$AB = BA$$

whenever

$$A \in \mathfrak{A}(\mathcal{O}_1), B \in \mathfrak{A}(\mathcal{O}_2),$$

and

$$\mathcal{O}_1, \mathcal{O}_2$$

are spacelike separated.

RUDOLF HAAG'S FAR REACHING VIEW (mid Fifties): CENTRAL PRINCIPLE OF QFT. **Experiments**: OK at all accessible scales; **Theory:** in QFT OK at all scales, ONLY if we neglect GRAVITATIONAL FORCES BETWEEN ELEMENTARY PARTICLES.

(Rudolf's intuition, $\simeq$ 80ies: otherwise, $\mathfrak{A}(\mathcal{O})$ ought to be **irreducible**!)

If we DON'T neglect Gravity:

Heisenberg: localizing an event in a small region costs energy **(QM)**;

Einstein: energy generates a gravitational field **(CGR)**.

QM + CGR:

PRINCIPLE OF *Gravitational Stability against localization of events* [DFR, 1994, 95]:

The gravitational field generated by the concentration of energy required by the Heisenberg Uncertainty Principle to localize an event in spacetime

should not be so strong to hide the event itself to any distant observer - distant compared to the Planck scale.

Spherically symmetric localization in space with accuracy a: an uncontrollable energy E of order $1/a$ has to be transferred;

Schwarzschild radius $R \simeq E$

(in universal units where $\hbar = c = G = 1$).

Hence we *must* have that

$$a \gtrsim R \simeq 1/a;$$

so that

$$a \gtrsim 1,$$

i.e. in CGS units

$$a \gtrsim \lambda_P \simeq 1.6 \cdot 10^{-33} cm. \qquad (1)$$

(J.A.Wheeler? . . . ? FOLKLORE).

But elaborations in two significant directions are surprisingly recent.

First, if we consider the energy content of a generic quantum state where the location measurement is performed, the bounds on the uncertainties should also depend upon that energy content.

Second, if we consider generic uncertainties, the argument above suggests that they ought to be limited by **uncertainty relations**.

To the first point:

background state: spherically symmetric distribution,

total energy E within a sphere of radius R, with $E < R$.

If we localize, in a spherically symmetric way, an event at the origin with space accuracy a, heuristic argument as above shows that

$$a \gtrsim (E - R)^{-1}$$

Thus, if $R - E$ is much smaller than 1, the “minimal distance” will be much larger than 1.

Quantum Spacetime can *solve* the Horizon Problem.

To the second point:

if we measure **one or at most two** space coordinates with great precision a, but the uncertainty L in another coordinates is **large**,

the energy $1/a$ may spread over a region of size L, and

generate a gravitational potential that **vanishes everywhere** as $L \to \infty$

(provided a, as small as we like but non zero, remains constant).

This indicates that the Δq^μ must satisfy **UNCERTAINTY RELATIONS.**

Should be implemented by **commutation relations**.

QUANTUM SPACETIME.

Dependence of Uncertainty Relations, hence of Commutators between coordinates, upon background quantum state i.e. **upon metric tensor.**

CGR: **Geometry** $\sim$ **Dynamics**

QG: **Algebra** $\sim$ **Dynamics**

Quantum Minkowski Space

The Principle of *Gravitational Stability against localization of events* implies :

$$\Delta q_0 \cdot \sum_{j=1}^{3} \Delta q_j \gtrsim 1; \sum_{1 \leq j < k \leq 3} \Delta q_j \Delta q_k \gtrsim 1. \qquad (2)$$

Comments:

- Derived [DFR 1994 - 95] using the *linearized* **approximation** to EE, BUT [TV 2012]: adopting the Hoop Conjecture a stronger form follows from an *exact* treatment, which applies to a curved background as well.

[DMP 2013]: special case of spherically symmetric experiments, with all spacetime uncertainties taking all the same value, the **exact semiclassical EE**, without any reference to energy, implies a **MINIMAL COMMON VALUE** of the uncertainties (of the minimal proper length) of order of the Planck length.

STUR must be implemented by SPACETIME commutation relations

$$[q_\mu, q_\nu] = i\lambda_P^2 Q_{\mu\nu} \tag{3}$$

imposing **Quantum Conditions** on the $Q_{\mu\nu}$.

SIMPLEST solution:

$$[q^{\mu}, Q^{\nu,\lambda}] = 0; \tag{4}$$

$$Q_{\mu\nu} Q^{\mu\nu} = 0; \tag{5}$$

$$((1/2)\,[q_0, \ldots, q_3])^2 = I, \tag{6}$$

where $Q_{\mu\nu} Q^{\mu\nu}$ is a scalar, and

$$
\begin{aligned}
[q_0, \ldots, q_3] &\equiv \det \begin{pmatrix} q_0 & \cdots & q_3 \\ \vdots & \ddots & \vdots \\ q_0 & \cdots & q_3 \end{pmatrix} \\
&\equiv \varepsilon^{\mu\nu\lambda\rho} q_\mu q_\nu q_\lambda q_\rho = \\
&= -(1/2) Q_{\mu\nu} (*Q)^{\mu\nu} \qquad (7)
\end{aligned}
$$

is a pseudoscalar, hence we use the square in the Quantum Conditions.

Basic model of Quantum Spacetime; **implements exactly Space Time Uncertainty Relations** and is fully **Poincaré covariant**.

The *classical* **Poincaré group acts as symmetries**; translations, in particular, act adding to each q_μ a real multiple of the identity.

The *noncommutative* C* algebra of Quantum Space-time can be associated to the above relations. The procedure [DFR] applies to more general cases.

Assuming that the $q_\lambda, Q_{\mu\nu}$ are selfadjoint operators and that the $Q_{\mu\nu}$ commute *strongly* with one another and with the q_λ, the relations above can be seen as a bundle of Lie Algebra relations based on the joint spectrum of the $Q_{\mu\nu}$.

Regular representations are described by representations of the C* group algebra of the unique simply connected Lie group associated to the corresponding Lie algebra.

The C* algebra of Quantum Spacetime is the C* algebra of a continuos field of group C* algebras based on the spectrum of a commutative C* algebra.

In our case, that spectrum - the joint spectrum of the $Q_{\mu\nu}$ - is the manifold Σ of the real valued antisymmetric 2 - tensors fulfilling the same relations as the $Q_{\mu\nu}$ do: a homogeneous space of the proper orthocronous Lorentz group, identified with the coset space of $SL(2, C)$ mod the subgroup of diagonal matrices. Each of those tensors, can be taken to its rest frame, where the electric and magnetic parts $\mathbf{e}$, $\mathbf{m}$ are **parallel unit vectors**, by a boost, and go back with the inverse boost, specified by **a third vector, orthogonal to those unit vectors**; thus Σ can be viewed as the tangent bundle to two copies of the unit sphere in 3 space - its **base** Σ_1.

Irreducible representations at a point of Σ_1: **Shroedinger** p, q **in** 2 **d. o. f.**.

The fibers, with the condition that I is not an independent generator but is represented by I, are the C* algebras of the Heisenberg relations in 2 degrees of freedom - the algebra of all compact operators on a fixed infinite dimensional separable Hilbert space.

The continuos field can be shown to be trivial. Thus the C* algebra $\mathcal{E}$ of Quantum Spacetime is identified with the tensor product of the continuous functions vanishing at infinity on Σ an the algebra of compact operators. It describes representations of the q_μ which obey the Weyl relations

$$e^{ih_\mu q^\mu} e^{ik_\nu q^\nu} = e^{-\frac{i}{2} h_\mu Q^{\mu\nu} k_\nu} e^{i(h+k)_\mu q^\mu}, \quad h, k \in \mathbb{R}^4.$$

The mathematical generalization of points are pure states.

Optimally localized states: those minimizing

$$\Sigma_\mu (\Delta_\omega q_\mu)^2;$$

minimum $= 2$, reached by states concentrated on Σ_1, at each point **ground state of harmonic oscillator**.

(Given by an **optimal localization map** composed with a probability measure on Σ_1).

But to explore more thoroughly the Quantum Geometry of Quantum Spacetime we must consider $independent$ $events$.

Quantum mechanically n independent events ought to be described by the $n - fold$ tensor product of $\mathcal{E}$ with itself; considering arbitrary values on n we are led to use the direct sum over all n.

If A is the C* algebra with unit over $\mathbb{C}$, obtained adding the unit to $\mathcal{E}$, we will view the $(n+1)$ tensor power $\Lambda_n(A)$ of A over $\mathbb{C}$ as an A-bimodule with the product in A,

$$a(a_1 \otimes a_2 \otimes ... \otimes a_n) = (aa_1) \otimes a_2 \otimes ... \otimes a_n;$$
$$(a_1 \otimes a_2 \otimes ... \otimes a_n)a = a_1 \otimes a_2 \otimes ... \otimes (a_n a);$$

and the direct sum

$$\Lambda(A) = \bigoplus_{n=0}^{\infty} \Lambda_n(A)$$

as the A-bimodule tensor algebra,

$$(a_1 \otimes a_2 \otimes ... \otimes a_n)(b_1 \otimes b_2 \otimes ... \otimes b_m) = a_1 \otimes a_2 \otimes ... \otimes (a_n b_1) \otimes b_2 \otimes ... \otimes b_m.$$

This is the natural ambient for the *universal differential* *calculus*, where the differential is given by

$$d(a_0 \otimes \cdots \otimes a_n) = \sum_{k=0}^{n} (-1)^k a_0 \otimes \cdots \otimes a_{k-1} \otimes I \otimes a_{\otimes} \cdots \otimes a_n.$$

As usual d is a **graded differential**, i.e., if $\phi \in \Lambda(A), \psi \in \Lambda_n(A)$, we have

$$d^2 = 0;$$
$$d(\phi \cdot \psi) = (d\phi) \cdot \psi + (-1)^n \phi \cdot d\psi.$$

Note that $A = \Lambda_0(A) \subset \Lambda(A)$, and the d-stable subalgebra $\Omega(A)$ of $\Lambda(A)$ generated by A is the *universal differential algebra*. In other words, it is the subalgebra generated by A and

$$da = I \otimes a - a \otimes I$$

as a varies in A.

In the case of n independent events one is led to describe the spacetime coordinates of the $j-th$ event by $q_j = I \otimes ... \otimes I \otimes q \otimes I ... \otimes I$ (q in the j - th place); in this way, the commutator between the different spacetime components of the q_j would depend on j.

A better choice is to require that it does not; this is achieved as follows.

The centre Z of the multiplier algebra of $\mathcal{E}$ is the algebra of all bounded continuos complex functions on Σ; so that $\mathcal{E}$, and hence A, is in an obvious way a $Z-bimodule$.

We therefore can, and will, replace, in the definition of $\Lambda(A)$, the $\mathbb{C}$ - tensor product by the $Z-bimodule-tensor$ $product$ so that

$$dQ = 0.$$

As a consequence, the q_j and the $2^{-1/2}(q_j - q_k)$, j different from k, and $2^{-1/2}dq$, obey **the same space-time commutation relations,** as does the normalized barycenter coordinates, $n^{-1/2}(q_1 + q_2 + ... + q_n)$; and the latter commutes with the difference coordinates.

These facts allow us to define a *quantum diagonal map* from $\Lambda_n(\mathcal{E})$ to $\mathcal{E}_1$ (the restriction to Σ_1 of $\mathcal{E}$),

$$E^{(n)} : \mathcal{E} \otimes_Z \cdots \otimes_Z \mathcal{E} \longrightarrow \mathcal{E}_1$$

which factors to that restriction map and a conditional expectation which leaves the functions of the barycenter coordinates alone, and evaluates on functions of the difference variables the *universal optimally localized map* (which, when composed with a probability measure on Σ_1, would give the generic optimally localized state).

Replacing the classical diagonal evaluation of a function of n arguments on Minkowski space by the *quantum diagonal map* allows us to define the *Quantum Wick Product*.

But working in $\Omega(A)$ as a subspace of $\Lambda(A)$ allows us to use two structures:

- the tensor algebra structure described above, where both the A bimodule and the Z bimodule structures enter, essential for our reduced universal differential calculus;

- the pre - C* algebra structure of $\Lambda(A)$, which allows us to consider, for each element a of $\Lambda_n(A)$, its modulus $(a^*a)^{1/2}$, its spectrum, and so on.

In particular we can study the geometric operators: separation between two independent events, area, 3 - volume, 4 - volume, given by

$$dq;$$

$$dq \wedge dq;$$

$$dq \wedge dq \wedge dq;$$

$$dq \wedge dq \wedge dq \wedge dq,$$

where, for instance, the latter is given by

$$V = dq \wedge dq \wedge dq \wedge dq =$$

$$\epsilon_{\mu\nu\rho\sigma} dq^{\mu} dq^{\nu} dq^{\rho} dq^{\sigma}.$$

Each of these forms has a number of spacetime components:

e.g. 4 the first one (a vector), 1 the last one (a pseudoscalar).

Each component is a normal operator;

$$THEOREM$$

For each of these forms, **the sum of the square moduli of all spacetime components is bounded below by a multiple of the identity of unit order of magnitude**.

Although that sum is (except for the 4 - volume!) NOT Lorentz invariant, the bound holds in any Lorentz frame.

In particular,

- the four volume operator has pure point spectrum, at distance $5^{1/2} - 2$ from 0;

- the *Euclidean* distance between two independent events has a lower bound of order one in Planck units.

Two distinct points **can never merge to a point**.

However, of course, the state where the minimum is achieved will depend upon the reference frame where the requirement is formulated.

(The structure of length, area and volume operators on QST has been studied in full detail [BDFP 2011]).

Thus the existence of a minimal length is not at all in contradiction with the Lorentz covariance of the model.

In the C* algebra $\mathcal{E}$ of Quantum Spacetime, define [DFR 1995]:

- the **von Neumann functional calculus**: for each $f \in \mathcal{F}L^1(\mathbb{R}^4)$ the **function $f(q)$ of the quantum coordinates q_μ** is given by

$$f(q) \equiv \int \check{f}(\alpha) e^{i q_\mu \alpha^\mu} d^4\alpha \ ,$$

- the **integral over the whole space** and **over 3 -**

space at $q_0 = t$ by

$$\int d^4q f(q) \equiv \int f(x) d^4x = \check{f}(0) = Tr f(q),$$
$$\int_{q_0=t} f(q) d^3q \equiv \int e^{ik_0t} \check{f}(k_0, \vec{0}) dk_0 =$$
$$= \lim_m Tr(f_m(q)^* f(q) f_m(q))),$$

where the trace is the ordinary trace at each point of the joint spectrum Σ of the commutators, i.e. a $\mathcal{Z}$ valued trace.

But on more general elements of our algebra both maps give Q - dependent results.

Important to define the interaction Hamiltonian to be used in the Gell'Mann Low formula for the S - Matrix.

Current investigations (D. Bahns, K. Fredenhagen, G. Piacitelli, S.D.) concern models of QST where the algebra of the commutators $[q, q]$ is not central or not even abelian.

QFT on QST

The geometry of Quantum Spacetime and the **free field theories** on it are *fully Poincaré covariant*.

One can introduce interactions in different ways, all preserving spacetime translation and space rotation covariance, all equivalent on ordinary Minkowski space, providing inequivalent approaches on QST; but all of them, sooner or later, meet **problems with Lorentz covariance**, apparently due to the nontrivial action of the Lorentz group on the *centre* of the algebra of Quantum Spacetime.

On this points in our opinion a deeper understanding is needed.

For instance, the interaction Hamiltonian on quantum spacetime

$$\mathcal{H}_I(t) = \lambda \int_{q^0=t} d^3q \ \ :\phi(q)^n:$$

would be Q - dependent; **no invariant probability measure or mean** on Σ; integrating on Σ_1 [DFR 1995] breaks Lorentz invariance.

Covariance is preserved by **Yang Feldmann equations** but missed again at the level of scattering theory.

The **Quantum Wick product** selects a special frame from the start. The interaction Hamiltonian on the quantum spacetime is then given by

$$\mathcal{H}_I(t) = \lambda \int_{q^0=t} d^3q \ \ :\phi(q)^n :_Q$$

where

$$:\phi^n(q):_Q = E^{(n)}(:\phi(q_1)\cdots\phi(q_n):)$$

which does not depend on Q any longer, but brakes Lorentz invariance at an earlier stage

The last mentioned approach takes into account, in the very definition of Wick products, the fact that in our Quantum Spacetime n (larger or equal to two) distinct points can never merge to a point. But we can use the canonical *quantum diagonal map* which associates to functions of n independent points a function of a single point, evaluating a conditional expectation which on functions of the differences takes a numerical value, associated with the minimum of the Euclidean distance (**in a given Lorentz frame!**).

The **"Quantum Wick Product"** obtained by this procedure leads to an interaction Hamiltonian on the quantum spacetime given by as a constant operator–valued function of Σ_1 (i.e. $\mathcal{H}_I(t)$ is formally in the tensor product of $\mathcal{C}(\Sigma_1)$ with the algebra of field operators).

The interaction Hamiltonian on the quantum spacetime is then given by

$$\mathcal{H}_I(t) = \lambda \int_{q^0=t} d^3q \ :\phi(q)^n:_Q$$

This leads to a unique prescription for the interaction Hamiltonian on quantum spacetime. When used in the Gell'Mann Low perturbative expansion for the S - Matrix, this gives the same result as the **effective non local Hamiltonian** determined by the kernel

$$\exp\left\{ -\frac{1}{2}\sum_{j,\mu} a_j^{\mu 2}\right\}\delta^{(4)}\left(\frac{1}{n}\sum_{j=1}^{n} a_j\right).$$

The corresponding perturbative Gell'Mann Low formula is then **free of ultraviolet divergences** at each term of the perturbation expansion [BDFP 2003] .

However, those terms have a meaning only after a sort of adiabatic cutoff: the coupling constant should be changed to a function of time, rapidly vanishing at infinity, say depending upon a cutoff time T.

But the limit $T \to \infty$ is difficult problem, and there are indications it does not exist.

A major open problems is the following.

Suppose we apply this construction to the renormalized Lagrangean of a theory which is renormalizable on the ordinary Minkowski space, with the counter terms defined by that ordinary theory, and with finite renormalization constants depending upon both the Planck length λ_P and the cutoff time T.

Can we find a natural dependence such that in the limit $\lambda_P \to 0$ and $T \to \infty$ we get back the ordinary renormalized Gell-Mann Low expansion on Minkowski space?

This should depend upon a suitable way of performing a joint limit, which hopefully yields, for the physical value of λ_P, to a result which is essentially independent of T within wide margins of variation; **in that case**, that result could be taken as source of **predictions to be compared with observations**.

Comments on QST and cosmology

Heuristic argument we started with: commutators between coordinates ought to depend on $g_{\mu,\nu}$; scenario:

$$R_{\mu,\nu} - (1/2)Rg_{\mu,\nu} = 8\pi T_{\mu,\nu}(\psi);$$

$$F_g(\psi) = 0;$$

$$[q^\mu, q^\nu] = iQ^{\mu,\nu}(g);$$

Algebra is Dynamics.

Expect: dynamical minimal length.

In particular, divergent near singularities. Would solve Horizon Problem, without inflationary hypothesis.

How solid are these heuristic arguments?

Exact semiclassical EE, spherically simmetric case: minimal proper length is at least λ_P [DMP, 2013].

Suggests:

massless scalar field semiclassical coupling with gravity;

use Quantum Wick product to define Energy - Momentum Tensor $T_Q^{\mu,\nu}(q)$;

Exact EE with source $\omega \otimes \eta_x(T_Q^{\mu,\nu}(q))$, where ω is a KMS state and η_x is a state on $\mathcal{E}$ optimally localized at x;

these simplifying *ansaetze* imply a solution describing spacetime without the horizon problem [DMP 2013]. Near the Big Bang every pair of points were in causal contact, as indicated by the heuristic argument that the range of a-causal effects should diverge.

Work in progress (Morsella, Piacitelli, Pinamonti, Tomassini, . . . , SD): search better grounds for the use of the Quantum Wick Product on curved spacetime.

Also: possible emergence of an effective inflationary potential as an effect of Quantum Spacetime.

Problem of the Cosmological Constant.

These would be new explanations of known scenarios.
Any new phenomenon?

QST: where to look for its shadows?

Not Lorentz violations, unless physical reasons impose them in special frames (rest system of CMB?).

But:

- Free Classical Electromagnetic Fields are not free on QST;

(discussions with Klaus Fredenhagen, mid nineties: energy loss passing a partially reflecting mirror evaluated to a fraction 10^{-130}).

- **Neutral particles are not free of em interactions on QST**.

Indeed this scenario was independently studied by other Authors:

R. Horvat, D. Kekez, P. Schupp, J. Trampetic, J. You: "Photon-neutrino interaction in theta-exact covariant noncommutative field theory" arXiv:1103.3383

Work in progress with Klaus Fredenhagen, Gerardo Morsella and Nicola Pinamonti:

- Is Dark Matter really dark? A pale quantum - gravitational moonshine might be possible.

On QST a $U(1)$ Gauge theory becomes a $U(\infty)$ theory, with gauge group (at each point of Σ) $\mathcal{U}(\mathbb{C}I + \mathcal{K})$, and neutral scalar fields are no longer gauge invariant, i.e. em interaction does not vanish: covariant derivative:

$$D_\mu \varphi(q) := \partial_\mu \varphi(q) - ie[A_\mu(q), \varphi(q)], \tag{8}$$

yields to a minimal interaction where a constant magnetic field B couples to a scalar neutral field as

$$L_I = \frac{e}{2}\epsilon_{jkh} B^j Q^{k\mu}(\{\partial_\mu \phi(q), \partial^h \phi(q)\}.$$

Remark: other choices of the covariant derivative must be possible - problem of the fractional charge of Quarks.

The expectation value of the right hand side in a state Φ can be used as an indication of the order of magnitude of the magnetic moment of neutral matter in that state, arising from QST.

Choose a constant magnetic field along the third axis and specialize to a point in the spectrum of the Q 's, where $\mathbf{e} = \mathbf{m} = \mathbf{e_1}$. The above term corresponds to the energy density of a magnetic moment with components

$$M_j = (e/2)\lambda_P^2 \int_{q^0=t} d^3q \, \left[\frac{1}{2}(\{\partial_l, \partial^l\}_{jk} - \{\partial_j, \partial^k\}) -_{jkh} \{\partial_0, \partial^h\} \right] e_k,$$

whose expectation value in a state Φ can be written as

$$(\Phi, M_j\Phi) = (e/2)\lambda_P^2\Lambda_j,$$

and can be used to give order of magnitudes. In a state Φ which describes a precession of the magnetic moment m with angular speed ω, according to Classical Maxwell Theory, the electromagnetic energy radiated per unit time is given by

$$(2/3)c^{-3}\omega^4 m^2,$$

so that in our case, neglecting constants of order one and setting $c = 1$, it would be

$$d\mathcal{E}/dt = e^2(\lambda_P\omega)^4\Lambda^2 = e^2(\tau_P/T)^4\Lambda^2$$

where $\Lambda^2 = \Lambda_1^2 + \Lambda_2^2 + \Lambda_3^2$, and T is the revolution period of the precession.

What if Φ describes an astrophysical object composed of dark matter, in rapid precession, as in the collapse of a binary system?

This might possibly give a seizable effect if T were of Planckian order, which would probably mean that our object collapsed into a black hole and no radiation is visible.

Can otherwise Λ be large enough? A reasonable computation in significant situations, e.g. of fast rotating binary systems of dark matter stellar objects, would be quite interesting.

We fit numbers suggested by the GW150914 event.

For an object of the size of our sun and water density, composed of particles of mass $= 1 GeV$, $T = 10^{-2} sec$, $R = 10^3 km$, a very rough estimate suggests that the fraction of the total energy which is emitted per second as electromagnetic radiation is of the order of

$$10^{-89}.$$

However, recall: QST suggests that near singularities the *effective Planck length* might diverge.

Results in [DMP 2013] mean in particular that in a flat Friedmann- Robertson-Walker (FRW) background (which is spherically symmetric with respect to every point), with metric, in spatial spherical coordinates, $ds^2 = -dt^2 + a(t)^2[dr^2 + r^2 dS^2]$, the size of a localisation region centered around an event at cosmological time t, measured by the radial coordinate r, must be at least of order $\lambda_P a(0)/a(t)$ (EFFECTIVE PLANCK LENGTH).

Could this make the em radiation, caused by the magnetic moment of neutral matter, considerably larger?

Note that the metric of a collapsing homogenous sphere of dust is given by the Oppenheimer-Snyder solution, which is a Schwarzschild metric outside the sphere, matched with a closed FRW metric inside.

This continuous match makes our antsatz less arbitrary.

In other words λ_P is replaced by $\lambda_P a(0)/a(t)$, and a rough estimate of the total energy $\mathcal{E}_r$ radiated by a neutral object precessing in the gravitational field of a collapsing object (as might be the case of some binary systems) would be given by the integral

$$\mathcal{E}_r = \int_{-\infty}^{collapse} e^2 (\tau_P a(0)/T(t) a(t))^4 \Lambda^2 dt$$

where:

1. We first have to average over Σ_1.

2. The time integration extends to the time when the second object is trapped within the event horizon of the first.

For, the em radiation emitted later would not reach distant observers.

As an indication we can extend the integration to the time when the radius of the collapsing object becomes the Schwarzschild radius.

3. The time dependence of T should take into account the energy emitted by the precessing object, at the cost of its kinetic energy.

4. Eventually, recalling that initially the metric is the Friedman Robertson Walker metric describing the interior of a collapsing star, we set $a(0) = \sqrt{R_0^3/2GM}$.

With M the ADM mass of the collapsing sphere, $R_0 \geq 2M$ the initial areal radius, the mass of the precessing object $\simeq M$, taking into account the smallness of λ_P^4, we obtain for the total radiated energy

$$\mathcal{E} \simeq e^2(\lambda_P a(0))^4 M^2 R^2 \omega_0^6 \left(\frac{2GM}{R_0^3}\right)^{3/2} F(\eta_{2M}),$$

where

$$\eta_{2M} = \cos^{-1}(4GM/R_0 - 1)$$

is the conformal time at which the sphere is completely inside its Schwarzschild radius, and

$$F(\eta) := \int_0^\eta \frac{dx}{(1+\cos x)^3} = \frac{\sin\eta(6\cos\eta + \cos(2\eta) + 8)}{15(1+\cos\eta)^3}.$$

Thus we see that for $2GM/R_0 = 1, F(\eta_{2M})$ vanishes, as it should, since the collapse takes place at the beginning.

If $2GM/R_0$ is smaller than 1 but of that order, then $F(\eta_{2M})$ is also of the same order. We take

$$M \simeq M_0 \simeq 10^{56} GeV = 10^{37} M_P \simeq \mathcal{E}_0,$$

$$R_0 = 4GM \simeq 10^{37} M_P^{-1} \simeq 10^{-1}\, km,$$

so that, assuming again

$$R \simeq 10^3 km \simeq 10^{41} M_P^{-1}$$

$$T \simeq 10^{-2} s \simeq 10^{42} \tau_P,$$

we get

$$\boldsymbol{\mathcal{E} \simeq 10^{-96} \mathcal{E}_0 \simeq 10^{-40} \mathrm{GeV}},$$

i.e. the same order of magnitude as the previous rough estimate per unit time, if we take into account the time duration, $\simeq 10^{-7} sec$.

No indication of visible effects so far!

Possible moral: the effects of Quantum Spacetime, even at astrophysical scales, become relevant only when hidden within an event horizon, hence not accessible to observations. Only signatures: **those maybe left in the Big Bang**.

BUT:

* Here GR corrections to the emission of em radiation by a precessing magnetic moment have NOT been

studied. Also quantum corrections might well be important.

As mentioned, ω is NOT constant: the radiation is emitted at the cost of

- kinetic energy (rotation, precession, revolution); making $\omega(t)$ decreasing;

but also of:

- energy in the gravitational field of the companion object; making $\omega(t)$ increasing;

(to be computed; no significant corrections may be expected);

Other possibilities / aspects:

* A massive fast rotating astrophysical object of dark matter might hide a distant source both of photons and of charged particles; the latter beyond the gravitational deflection would feel the magnetic field due to the moment we discussed here, as the electrons causing northern light near the earth pole. This different behaviour might be a signature of QST.

* Self gravitating Bose - Einstein Condensates of NEUTRAL scalar particles (high density and high angular momentum)?

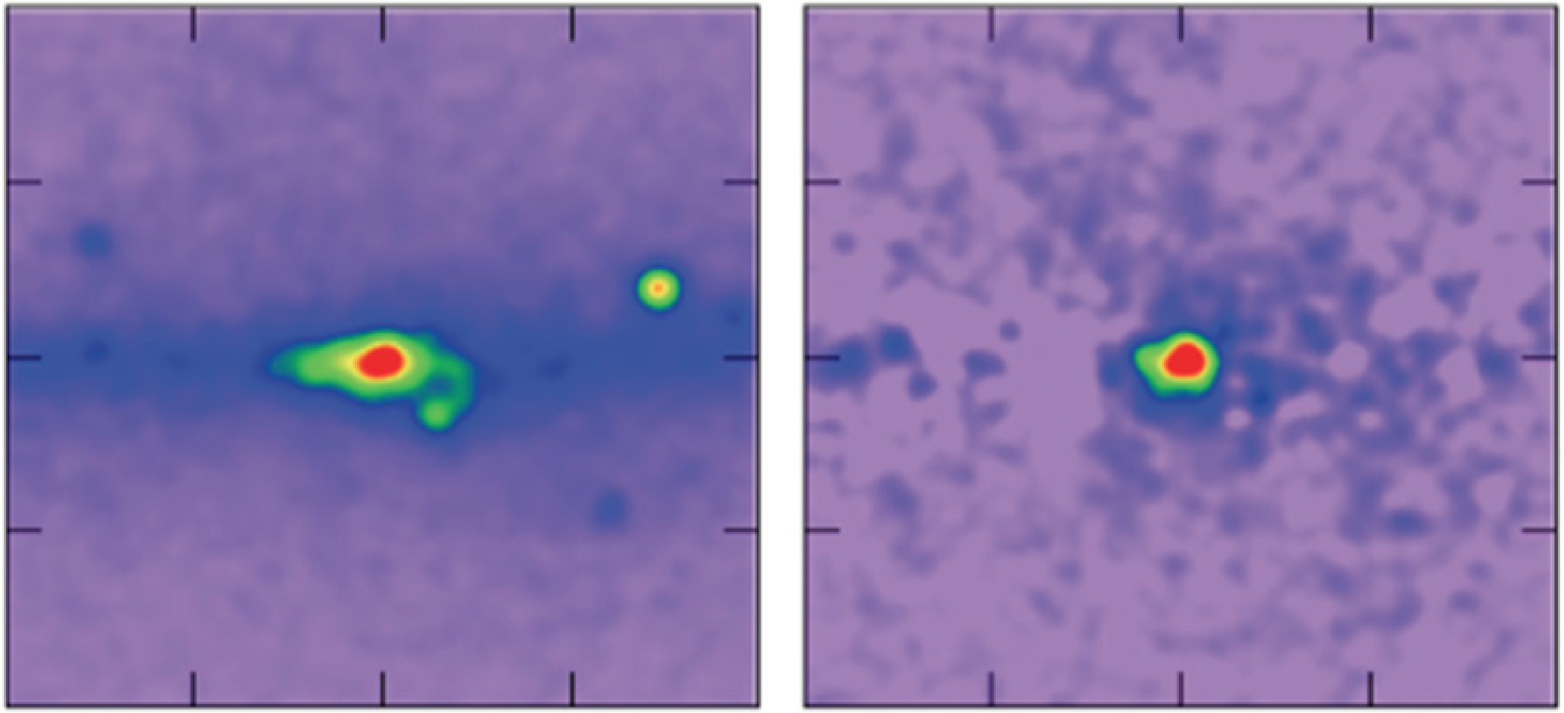
* Comparison with graviton mediated em interaction?

* Comparison with Hawking radiation?

(DFMP, work in project).

HOWEVER x ray emission from the BH at the center of the Milky way already points to a similar scenario:

2014 March 10



Gamma Rays from Galactic Center Dark Matter?
Image Credit: T. Daylan et al., Fermi Space Telescope, NASA

Warning: Quantum effects at Planck scale result from extrapolation of EE to that scale.

But: Newton's law is experimentally checked only for distances **not less than .01 centimeter!** (Adelberger *et al, 2003, 2004), i.e. we are extrapolating 31 steps down* **in base 10 - log scale***; while the size of the known universe is "only" 28 steps up.*

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