String-local renormalization theory, applications to Higgs models and QED

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(in part based on a joint project with K-H Rehren and J. Mund, arXiv:190609596)

Outline:

Kinematic prerequisites of SLF

Interactions m>0, S-matrix and sl Bogoliubov map

Extension to m=0, Gauß law and Infra-particles

Resumé and open problems

Kinematic prerequisites 1

- Helicity $h \geq 1$ Wigner repr. have no *physical* pl potentials, but curl $A = F$ has a unique covariant and *causally separating sl solution acting in the same Hilbert space as* F $(e^2 = -1)$

$$A_\mu(x,e) = \int_0^\infty F_{\mu\nu}(x+se)e^\nu ds, \;\; e^\mu A_\mu = 0, \;\; e\partial A_\mu = F_{\mu\nu}e^\nu$$

- Massive counterpart by "Fattening": $p \in V_+ \longrightarrow H_+$, $d\mu(p)_0 \longrightarrow d\mu(p)$ in 2ptfct

$$\langle A_\mu A'_\nu \rangle = \int e^{-ip(x-x')}(-g_{\mu\nu} + \frac{p_\mu e_\nu}{pe_{-i\varepsilon}} + \frac{p_\nu e'_\mu}{pe'_{i\varepsilon}} - \frac{p_\mu p_\nu}{pe_{-i\varepsilon}pe'_{i\varepsilon}})d\mu(p)_0$$

Sl 2ptfct permit smooth passing between the sl $h = \pm 1$ helicity fields and its massive 3-component spin counterpart in which *causal separability* is preserved.

Kinematic prerequisites 2

- A_μ together with the pl A^P_μ obeys the linear *Escort Relation:*

$$A_\mu(x,e) = A^P_\mu(x) + \partial_\mu\phi(x,e),\ \phi(x,e) = \int_0^\infty ds A^P_\mu(x+se))e^\mu$$

  proof: both A and its Proca counterpart A^P share F and hence differ by a gradient of ϕ which is a solution of the linear relation $e^\mu A_\mu = 0 = e^\mu A^P_\mu + e\partial\phi$.

- Looks like a gauge transformation in GT but *is not a symmetry*. The fields share the $a^\#(p,s_3)$ Wigner operators but differ in their intertwiners functions and their d_{sd} : $d_{sd}(A) = 1 = d_{sd}(\phi)$, $d_{sd}(A^P) = 2$. Even more: the hybrid connection between SLF and GT (see later) permits the extension of gauge invariant observbles to positivity-maintaining sl interpolating fields.

Kinematic prerequisites 3

- To derive the important theorems of QFT (TCP, Spin&Statistics,...) fields need not be pl but must remain *causally separable*. This (together with a mass gap) constitutes the prerequisite for the existence of the LSZ scattering matrix S on which the causal localization properties of fields leave their well-known mark in the form of analytic properties of scattering amplitudes (dispersion relations).
- SLF uses the e-independence of S as a *powerful restrictive tool for its perturbative construction* in terms of sl interaction densities L. Unlike GT the short distance bound of renormalizability $d_{sd}(L) \leq 4$ is achieved *without the use of positivity-violating unphysical degrees of freedom*.

Interactions m>0, S-matrix and sl Bogoliubov map 1

- How does this work? One first uses the escort equation $A = A^P + \partial\phi$ to lower the $d_{sd}(L^P) = 5$ to $d_{sd}(L) = 4$. The simplest illustration is provides by "massive QED" $L^P = A^P_\mu j^\mu$ with $\partial j = 0$. Defing its sl counterpart $\not{L}$ as

$$L = L^P + \partial_\mu V^\mu = A_\mu j^\mu, \;\; V_\mu = \phi j_\mu$$
$$\int g(x)\partial_\mu V^\mu(x) d^4x = -\int \partial_\mu g(x) V^\mu(x) d^4x \overset{g(x)\to g}{\longrightarrow} 0$$

  lowers $d_{sd}(L^P) = 5$ to $d_{sd}(L) = 4$. The vanishing V_μ part in the adiabatic limit is the precondition for the equality of the L^P S-matrix with that of L.

- For the equality of the S-matries

$$S = T \exp ig \int L(x,e) d^4x = T \exp ig \int L^P(x) d^4x \qquad (1)$$

  ∂_μ must commute with time-ordering: $\partial_\mu T..V^\mu.. = T\partial_\mu..V^\mu..$

Interactions m>0, S-matrix and sl Bogoliubov map 2

- Using the diff. form calculus on $d = 1 + 2$ de Sitter ($d_e = d_{e_\mu} \partial_e^\mu$, $Q^\mu = d_e V^\mu$, $d_e L - \partial Q = d_e L^P = 0$) this demands

$$d_e TL(x_1, e)..L(x_n, e) = \sum_i \partial_{\mu_i} T(L..Q^{\mu_i}(x_i, e)..L) \qquad (2)$$

- Fulfilled in spinor QED, but violated in *scalar* QED by the kinematical Feynman propagator $T_0 : \langle T_0 \partial_\mu \varphi^* \partial'_\nu \varphi' \rangle = \partial_\mu \partial'_\nu \langle T_0 \varphi^* \varphi' \rangle$. Way out: use the (scaling degree-preserving) renorm. freedom and replace $T_0 \rightarrow T$ by adding $icg_{\mu\nu}\delta(x - x')$ with $c = -1$

Interactions m>0, S-matrix and sl Bogoliubov map 3

- Going back to T_0 this amounts to

$$TL(x_1,e)L(x_2,e) = T_0L(x_1,e)L(x_2,e) + \delta(x_1 - x_2)\,\frac{1}{2}A_\mu A^\mu \varphi^* \varphi \quad (3)$$

  which in 2^{nd} order contribution, which in GT follows from gauge invariance, is here a consequence of the causality and positivity principles of QFT which are the basis of SLF.

- In Higgs type models the construction of the first order L, V_μ pair from $d_e(L - \partial V) = 0$ is much more elaborate. There are 20 potential contributions to L of products of maximally 4 fields namely $AA\partial A, AA\phi, A\partial\phi\phi, \phi^3; AAH, A\partial\phi H, A\partial H\phi, \phi^2 H; A\partial HH, \phi H$ $AA\phi^2, \phi^4;\ AA\phi H, \phi^3 H, AAH^2, \phi^2 H^2; \phi H^3, H^4$. and there also a large number of potential contributions to V_μ. Using the $gAAH$ as the "spark-igniting" for a A-H interaction with coupling strength g, the L, V_μ pair requirement eliminates most contributions.

Interactions m>0, S-matrix and sl Bogoliubov map 4

- Apart from H selfinteractions (whose a, b coupling strength remains undetermined the first order) the result is unique and has the following form

$$L(x,e) = A(AH - \frac{1}{2}\phi^2 \overleftrightarrow{\partial} H) + \frac{m_H^2}{2}\phi^2 H + aH^3 + bH^4$$

$$V_\mu(x,e) = A_\mu^P \phi H + \frac{1}{2}\phi^2 \partial_\mu H, \quad A_\mu^P = A_\mu - \partial_\mu \phi$$

- In second order the change of time-ordering is not sufficient, there remain a genuine *obstructions* of form $\delta(x-x')L_2(x,e)$. Solution: use $L_{tot} = L + \frac{g}{2}L_2$ in Bogol. S-matrix formula. The L_2 is referred to as an "induced" contribution.

- The requirement of absence of 3rd order obstructions fixes $a=-\frac{m_H^2}{2m^2},\ b=-g\frac{m_H^2}{4m^4}$ so that H^3 contributes to $1st$ and H^4 to $2nd$ order. The net result up to second order is

$$L_{tot}=A(AH-\frac{1}{2}\phi^2\overleftrightarrow{\partial}H)+\frac{m_H^2}{2}\phi^2H-\frac{m_H^2}{2m^2}H^3$$
$$-g[\frac{m_H^2}{4}\phi^4+\frac{m_H^2}{2m^2}\phi^2H^2+\frac{m_H^2}{4m^4}H^4]$$

  The presence of H selfinteractions bears no conceptual relation to a *postulated Mexican hat potential* for the purpose of SSB.
- A similar calculation for *selfinteracting* vector potentials reveals that the A_μ selfcouplings have a *Lie-algebra structure.* This is surprising since unlike GT, SLF is exclusively based on spacetime causality properties. They are also the only known models which *require an enlargement of their particle content by an Hermitian scalar field H. This (i.e. the fundamental SLF principles and not SSB) is the raison d'être for the Higgs.*

A quick summary and a lookahead

- The preservation of the power-counting bound of renormalizability $d_{sd}(L) \leq 4$ in the Bogoliubov construction of S requires the use of $s = 1$ free fields in the form of line integrals $A_\mu(x, e)$, whereas $s < 1$ fields enter L as pl free fields. The L, V_μ pair condition and its higher order extension fixes L_{tot} in terms of the model's particle content.
- Unexpected changes occur in the construction of interacting fields in terms of the *Bogoliubov map,* which relates free fields with their *interacting* counterpart. In the presence of sl $s = 1$ the result takes the form of a dichotomy between pl *observables* and *sl interpolating fields*. In particular all interacting $s < 1$ fields (including H) which enter S as pl free fields become genuine sl interpolating fields. Unlike line integrals (which can be inverted to pl by -$e\partial$) genuine interpol. fields cannot be re-converted into pl fields in terms of local operations.

Interactions m>0, S-matrix and sl Bogoliubov map 6

- Interacting fields are defined in terms of the adiab. limit of the Bogol. operator S-functional, formally

$$B|_L = S^{-1}(gL)\frac{\delta}{\delta f(x)}S(gL+fB)|_{f=0} \quad (4)$$

- $B \rightarrow B|_L$ converts Wick-ordered fields B from the free pl Borchers class (corresponding to the particle content of the models) into their interacting "normal-ordered" interacting counterparts $B|_L$ (which take the form of a retarded perturbative expansion). The map changes the algebraic structure and localization but *preserves causal separability*. We will refer to all fields (pl or sl Wick-ordered composites) from the free Borchers class as "free fields".
- The Bogoliubov images of pl free fields fall into two groups separated by their localization properties: pl observables (as $F_{\mu\nu}$) and genuine sl interpolating fields (i.e. not line integrals over observables).

Interactions m>0, S-matrix and sl Bogoliubov map 7

- With $L = L^P + \partial V$ each $B|_L$ has also a L^P pre-image (source) $\hat{B}$ with
$$\hat{B}|_{L^P} = B|_L \tag{5}$$
$$\hat{B}(x,e) = B(x) + g\hat{B}_1(x,e) + g^2\hat{B}_2(x,e) + ..$$
where the $\hat{B}_i$ arise from terms of the form $\partial_\mu TV^\mu..B.. - T\partial_\mu V^\mu..B.. \neq 0$. In this calculation the shared *prefactor* S^{-1} in (4) can be *omitted* so that $B \rightarrow \hat{B}$ is a causal separability preserving map within the *sl-extended free Borchers equivalence class*; the $\hat{B}_i(x,e)$ represent higher order sl corrections to $B(x)$.
- This map permits to distinguish observables ($\hat{B}(x) = B(x)$) from interpolating fields ($\hat{B}(x,e) \neq B(x)$) already on the level of free source fields (i.e. without use of full-fledged renorm. theory. All elementary $s < 1$ free fields (which includes H) turn into interacting sl interpolating counterpart e.g. in massive QED one finds $\psi(x)|_L =: e^{ig(x,e)\phi}\psi(x) : |_{L^P}$. For composite fields the calculations are more involved

Interactions m>0, S-matrix and sl Bogoliubov map 8

- Note that the pl axiomatic Wightman setting is only compatible with perturbative interactions between of $s < 1$ particles (the old π-N interactions). The *dichotomy between interacting pl observables and sl interpolating fields* (the "yin and yang" of $s \geq 1$ QFT) reveals an unexpected tight connection between *quantum positivity and causal separability* in the presence of interactions which GT misses.
- Note that the original purpose of using $s = 1$ line integrals was to *preserve renormalizability of S*. The surprising dichotomy of interacting fields unfolds only in the second step when one uses the Bogoliubov map. The main theorems of Wightman QFT (TCP, Spin&Statistics,..) remain valid in SLF since their proof depends only causal separability. In the presence of $s \geq 1$ SLF replaces Lagrangian quantization by maintaining positivity (the probability interpretation of QFT. Unlike pl Lagrangian fields sl interpolating fields have no classical limit, only $s \geq 1$ (Maxwell, Einstein) observables have.

Interactions m>0, S-matrix and sl Bogoliubov map 9

- The A_μ line-integral plays important role in SLF Bogoliubov map. Without its presence in Lno $B \to \hat{B}$ and hence no dichotomy between pl observables and sl interpolating fields. In contrast to the $s < 1$ fields which are turned into genuine sl fields, A_μ remains a line in every integral since $F(x)|_{L^P} = F(x)|_{L(e)}$ and hence it remains a line-integrated observable in every order $A_\mu(x,e')|_{L^P} = A_\mu(x,e')|_{L(e)}$ (the directional derivative $e'\partial$ reconverts it into an observable) instead becoming a sl interpolating field. Its role in $L(e)$ is to maintain S renormalizable and e-independent.
- In the Bogoliubov transformation A_μ plays the role of a "catalyzer": it accounts for the dichotomic split while remaining itself a line integral over the observable F. This situation changes in the presence of *selfinteracting* vector-potentials $A_{\mu,a}$ which in addition "catalyze themselves" into genuine interpolating fields $(F_{\mu\nu,a}(x)|_L = \hat{F}_{\mu\nu,a}(x,e)|_{L^P},\ A_\mu(x,e)|_L = \hat{A}_\mu(x,e)|_{L^P})$.

- Without the "L-guidance" the L^P formalism is nonrenormalizable in a two-fold way ("strongly" nonrenormalizable) namely $d_{sd} \rightarrow \infty$ and the (with perturbative order) increasing number of counterterm parameters (and therefore without physical predictability). With L-guidance: renormalizable observables are shared with L^P (instead of $d_{sd} = \infty$ in the unguided L^P setting). Can this be understood in terms of compensations within the pl Epstein-Glaser formalism ?
- The L-guidance suggests that the remaining pl $d = \infty$ fields are "Jaffe fields" since they correspond (for say the ψ of massive QED) according to $\psi|_{L^P} = e^{-ig\phi}\psi|_L$ to the L-image of a free $d_{sd} = \infty$ Jaffe field. Question: are nonrenormalizable L^P fields Jaffe fields?
- Conjecture: Jaffe fields create compact localized states but do not lead to operator algebras (fail on domain properties). They may still provide Buchholz-Fredenhagen states on Haag-Kastler observable algebras.

Extension to m=0, Gauss-Law and Infraparticles 1

- SLF provides new insights into two related problems of pl QED: the absence of charge-carrying states $\psi\left|0\right\rangle$ with nontrivial Gauss charges $\langle Q\rangle \neq 0$

$$\partial^\mu F_{\mu\nu} = j_\nu, \quad \lim_{R\to\infty}\int \vec{E}d\vec{\sigma}_R = Q = \lim_{R\to\infty}\int_{|\mathbf{x}|\leq R} j_0(x)d^3x \qquad (6)$$

  With $[j_o(x)\psi(x')] \sim \delta(x-x')\psi(x')$ and $R\to\infty$ this excludes a pl ψ. Way out: use SLF in which higher order ψ become interpolating sl fields through contact with $A_\mu(x,e)$ in L. The charge Q is a central operator of the observable algebra.
- In addition to the Gauss law SLF accounts also for *asymptotic fluxes* $a_{\mu\nu}(x) = \lim_{\lambda\to\infty}\lambda^{-2}F_{\mu\nu}(\lambda x)$, $x^2 \neq 0$ which constitute a refinement of the GL; the central of observables $a_{\mu\nu}$ correspond to the (Lorentz symmetry-breaking) superselection sectors of spacelike "photon clouds". They are in turn related to the breakdown of the Wigner Fock particle Hilbert space and LSZ scattering theory.

Extension to m=0, Gauß-Law and Infraparticles 2

- For $e = (0, \vec{e})$ one obtains in lowest order ($q = charge$)

$$\lim_{\lambda \to \infty} \lambda^2 \left\langle \psi(f) \vec{E}(\lambda \vec{x}) \psi(f)^* \right\rangle = -q\vec{e} \int_0^\infty ds \delta(\vec{x} + s\vec{e}),$$

  so that the superselected asymptotic flux density in direction $\vec{x} = r\vec{e}$ (after averaging $L(x, e)$ with $h(\vec{e}) = 1$) is -$q\vec{e}h(e) = \frac{\vec{x}}{|\vec{x}|}q$.The result is expected to be independent of perturbation theory.
- To provide arguments for the nonpert. nature of this flux law and the connection with a new scattering theory of "infraparticles" one needs a different formulation of the the SLF pertubation theory which replaces the linear relation between $A_\mu(x)^L := A_\mu(x, e)$ and $A^P(x)$ by a linear "hybrid" relation between A^L and the pl A^K by embedding the $m > 0$ Wigner-Fock space (or its $m = 0$ helicity counterpart) as a factor space into a Gupta-Bleuler Krein space (i.e. no additional Stückelberg- and ghost- d.o.f. are needed!).

Extension to m=0, Gauss-Law and infraparticles 3

- The starting point of the hybrid setting is an identity between the *SLF* and a gauge setting (definition of ϕ^K in a shared Krein space)

$$A_\mu(x,e) = A_\mu^K(x) + \partial_\mu \phi^K, \ \phi^K(x,e) = -\int ds A_\mu^K(x+se)e^\mu$$
$$L^K(x) + \partial^\mu V_\mu^K(x,e) = L(x,e)$$

  where the second line is the relation between the corresponding interaction densities.

- The advantage is that both sides of the linear relation admit a $m=0$ limit. The ϕ^K by itself is logarithmically infrared-divergent and its use in calculations needs special attention.
- As in the case of the previous A^S-A^P relation hybrid massive QED leads for $m>0$ to $e^{ig\phi^K}\psi|_{L^K} = \psi|_{L^S}$.

Extension to m=0, Gauss-Law and infraparticles 4

- The 2-ptfct $\left\langle e^{-ig\phi^K(x,e)}\ e^{ig\phi^K(x',e')}\right\rangle$ is singular in the limit $m \to 0$. By an explicit (but somwhat tedious) calculation one can show that, after multiplying with a g-dependent power of m and the choice $e = e'$, one obtains a finite positive 2-ptfct. $\leq$The use of the limit states for expectation value of $F_{\mu\nu}$ extends the previous first result of the flux law to all orders.
- A similar much simpler infrared situation is met in the $d = 1 + 1$ explicitly solvable derivative coupling $gj_\mu\partial^\mu\phi$ of a Dirac current to the derivative of a massless scalar field for which $\psi_{infra} = \lim_{m\to o} m^{g^2/8\pi} \exp ig \int \partial_x\phi dx \cdot \psi$ converges. The resulting $\partial\phi$ flux in ψ_{infra} states have no e-dependence and the appropriatly defined massless limit of the corresponds "ϕ-clouds" in infraparticles states indicates the softening of the ψ mass shell and a corresponding faster $t \to \infty$ decrease $t^{-(1/2+g^2/4\pi)}$ than the kinemactic $t^{-1/2}$ LSZ decrease for massive ψ.

Extension to m=0, Gauß-Law and Infraparticles 5

- A rigorous derivation (based plausible assumptions about QED in the setting of AQFT) which relates the modification of the mass-shell singularity directly to the asymptotic flux had been previously established by Buchholz.
- The presence of the $\dot{g}$-dependent power of m including limit which led to the infraparticle structure in the $d = 1 + 1$ derivative model which led infraparticles (which extend Wigner particles). This also ensues a Lorentz symmetry breaking asymptotic $\partial\phi$ flux. This causes the vanishing of the LSZ limit and also explains the logarithmic on-shell divergencies in the momentum space LSZ formula (whose removal in terms of the BN or YFS soft photon resolution prescription lead to the Lorentz violating (but rotation preserving) prescription for scattering probabilies.
- The stronger $t \rightarrow \infty$ decrease can only be compensated by wave functions with a continuous mass distribution. The infraparticle Hilbert space results from a GNS construction in terms of vacuum expections of ψ_{infra}. This still remains to be done.

Resumé and open problems 1

- SLF is the only known positivity- and causal separability- preserving setting of perturbative QFT. It leads to significant conceptual changes as compared to Lagrangian quantization. It use in low orders leads to the presented results but its higher orders are presently not known.
- The unique determination of $s \geq 1$ SLF models in terms of their particle content (not true for $s < 1$) confers a more fundamental status to the Standard Model than that of a merely phenomenological description.
- The existence of conserved currents leading to *nonabelian* local symmetries in *interacting* SLF (viz. A_μ-selfinteractions) remains questionable since the Lie-algebraic structure of massive $A_{\mu a}(x, e)$ selfinteractions is not imposed by model-building physicists but rather a result of Nature's principle causality and positivity. The particle tables of Nature contain only p$\bar{\text{p}}$ particle doublets and no higher multiplets.

Resumé and open problems 2

- For $s = 2$ there are L, V_μ which fulfill the pair condition but unfortunately violate the power counting restriction $d_{sd}(L) \leq 4$ of renormalizability ($d_{sd}(L) = 5$). Can SLF keep the number of parameters finite?
- Nonabelian GT *in covariant gauges have no off-shell* infrared divergencies. Is this also the case in the somewhat infrared stronger SLF? If not, there may be chance to understand confinement in SLF.
- One urgently needs 4^{th} order calculations involving two sl A_μ internal propagators (box diagram) before setting out on a systematic sl renormalization which extends the pl Epstein-Glaser formalism.

Resumé and open problems 3

- Gauge bridges between pl positivity-violating charge-carrying fields free fields were previously used in the work by Steinmann, Morchio and Strocchi. As a heuristic tool for envisaging Haag duality for multiply localized double cone algebras they were used in Haag's book. Our motivation (Mund, Yngvason, Schroer 2005) started from the observation that the infinite spin Wigner reprs. maintain finite d_{sd} which opened their use of sl fields in propagators of renormalized perturbation.
  The $n = \infty$ Pauli-Lubanski limit of sl $A_{\mu_1..\mu_n}$ does not exist (Rehren) and there is no infinite spin GT gauge theory. This rules out interactions with lower spin matter (reactive inertness of infinite spin matter). The existence of such limits for composite tensor fields as e.g. the energy-momentum tensors permits however couplings to external potentials. Question: can couplings to external $g_{\mu\nu}$ lead to gravitational backreactions? Relation to dark matter?

The references relevant to this talk can be found in:
J. Mund, K.-H. Rehren and B. Schroer, Gauss Law and String-localized Quantum Fields, arXiv:1906.09596

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